PROCEEDINGS
THE INSTITUTION OF CIVIL ENGINEERS

PART I
MARCH 1952

ORDINARY MEETING

18 December, 1951

ALLAN STEPHEN QUARTERMAINE, C.B.E., M.C., B.Sc.,
President, in the Chair

The President said that he very much regretted to have to inform the Meeting of the recent death of Sir Peirson Frank, who had been President of The Institution during the year 1945–46. A Resolution of Condolence had been passed by the Council and conveyed to Lady Frank.

The Council reported that they had recently transferred to the class of

Members

Ronald Ernest Dudley Bain, B.E., B.A.I. (*Dublin*).
Kurt Billig.
James Norman McKay Campbell, B.Sc. (Eng.) (*Lond.*).
Cyril Cooper.
Russell Mackenzie Currie.
Walter Charles Duffill.
James Robbie Farquharson, C.B.E., B.Sc. (*Glas.*).
Trevethan Claude Frampton, O.B.E.
Wilfrid Esmond Hamilton, B.Sc. (Eng.) (*Lond.*).
Charles Kenneth Haswell, B.Sc. (Eng.) (*Lond.*).
Alexander Leslie, T.D., M.A. (*Cantab.*).
William Thomas Marshall, Ph.D., B.Sc. (Eng.) (*Lond.*).
Solomon Simon Morris, B.Sc. (*Cape Town*).
Henry Olivier, M.Sc. (*Cape Town*).
John Christopher Quinn, M.A., M.A.I. (*Dublin*).
Harry Ridehalgh.
Kyaw Sein, M.Sc. (*Manchester*).
Alan John Watkins.

and had admitted as

Students

Daniel Washington Ibiyemi Aluko.
Zaher Baher Abdalla, M.Sc. (Eng.).
Roderick Agnew.
Murtada Ahmed
Aftab Alam, B.Sc. (*Punjab*).
Maurice Graham Allinson.
Allan Peter Andrews.
John Bryan Armstrong.
John Shaw Aspinall.
James Humphreys Baines, B.A. (*Cantab.*).
Philip Dennis Baker.
George Keith Balding.
John Leonard Barrell.
Bryan Allen Bartley.
John Keith Bateman.
Arthur Roland Bates.
Derrick Matthew Beusmans.
William James Bevis (Jnr.).
Alan Wilson Blakey.
Christopher John Blundell.
Richard Thomas Alverne Bolitho.
Clive Bottrill, B.Sc. (*Manchester*).
Dennis Walter Albert Bower.
Peter Murray Boyd, B.Sc. (*Nottingham*).
Norbert Martin Braun.
John Anthony Broadhead.
Derek Clifford Brunning.
William Brooksbank.
Brian John Broomfield.
Peter Edmund Bull.
Alan Ernest Bullock.
Derek Hewitt Burnley.
Samuel James Callaghan.
Peter Bardolph Caplin.
Isaac Francis Carr.
Thomas Alexander Chalmers.
Michael Nicose Christodoulides.
Vincent Percival Chung.
Thomas Edward Clarkson.
John Cecil Clutterham.
Allan Boyne Cochran.
Ronald Colley.
Bertram Gerald Thorpe Copeland.
Bryan Copley.
Ian Corsie.
Donald Stanley Courten.
Cameron Craig.
John Norman Crow.
Neville Crowther.
Edward James Cunningham.
Edward Nathaniel Davies.
Michael Alan Gauntlett Dear.
Derek Geoffrey de Belder, B.Sc. (Eng.) (*Lond.*).
Ponnahennedige Nagesha Kamal Dias.
Alan Thomas Doel.
John Alexander Dolan.
Joseph Byrom Duffy.
Denis Nigel Warriner Earp.
John Michael Eddison.
Neville Bruce Ede.
Peter Edington Ellen.
Hugh Richard Maximilian Elliot, B.Eng. (*Liverpool*).
Michael John Essex.
Glyn Evans.
John Theodore Finey.
John Hugh Bennett Fisher.
Ronald Edmund Fitzgeorge.
Eric Fowles.
David John Munson French.
Donald William French.
John Michael Gallery.
John Alfred Gallon.
Ponnusamy Ganendra, B.Sc. (Eng.) (*Lond.*).
Ian Provan Gardiner.
Stewart Garrett.
Roy William Gittins.
John Glanville.
Edward Francis Glover, B.A (*Cantab.*).
James Lawrie Gordon.
Rachmiel Gorin.
Brian Gormley.
Peter John Grant.
Alexander Iain Grassie.
Maurice Greenbaum.
Alan Norman Grice.
Robert George Gussey.
Trevor David Hallworth.
Roy Joseph Halstead.
Douglas James Hamilton.
John Anthony Hannaford.
Maxwell Bryce Hart.
John Terence Crawford Harvey.
Walter Hay.
Neville Barrie Heathcote.
James Robert Heaver.
Ronald Alwyn Hesp.
Brian Stuart Hicklenton.
Barry Oliver Hilson.
Gerald Hinxman.
Robert Hodgson.
Anthony Ernest Holmes.
Raymond William Horn.
Norman Hoyle.
Joseph Brian Hughes
Alan John Rodney Hughesdon.
Ian Hutchison.
Geoffrey Hammond Hyde.
Lakshman Holm Iswariah.
Donald Grant Jefferies.

Robert Arthur Jones.
Arthur Denis Keene.
Roy William Kent.
Colin Harold Kingsbury.
Neville Jayampathie Bandaranayake Kotagama.
Vijita Sitisena Kulatunga.
John Howard Lea.
David Gadsby Lewis.
Graham Lewis.
Peter Light.
Eric Litton.
Peter Lownds, B.Sc. (*Manchester*).
Francis McCarvill.
Peter Macfarlane.
Noel Ian Murray McGill.
Robert McGowan.
Brian Robertson McKenna.
John Michael McKenna.
Ronald Dickson McMurray.
Thomas McGregor McNie.
Kumarasamy Mahesan.
Robert Brewster Markham.
Arnold Derek Middleton.
Leonard Minshull.
Christopher Kendall David Mitchell.
David Cameron Mollison.
Richard Burton Monk.
Donald Morgan.
John Kelland Morgan, B.Sc. (*Wales*).
Ronald Stewart Morison.
Derek Munro.
John Edward Nethaway, B.Sc. (Eng.) (*Lond.*).
Peter George Nicolson.
Mayer Nispel.
Thomas Akinola Jaiyeola Olufunmilayo Oluwole.
Peter George O'Neill
Maurice Percival Palfrey.
Peter Michael Palmer.
John Frederick Patrick.
Cyril John Penniket.
David Christopher Peters.
Rowland Francis Pocock.
Malcolm Thomas Frederick Pratt, B.Sc. (Eng.) (*Lond.*).
Christopher Pugh.
Denys Arthur Race, B.Sc. (Eng.) (*Lond.*).
Peter Reeves.
Karl Simmons Renner.
Gilbert Richardson.
George Leslie Robinson.
Derek Frederick Roe.
Vincent Anthony Ross.
Edward Campbell Ruddock, B.A., B.A.I. (*Dublin*).
David Augustine Sankey.
Paul Herbert Scarlett.
Gerard Seymour Malcolm Schokman.
Sebhat Hable Selassie.
Mahinda Ehelepola Seneviratne.
Robin Shepherd.
David William Smith.
Peter Donald Stephens.
Colin Bennett Stevens.
Hugh Walker Strain, B.A., B.A.I. (*Dublin*).
Roy Sutton.
Alan Edward Swann.
Alan Wylie Tatford.
Peter Taylor.
Robin Henry Thirlway.
David Thomas.
Alexander Keith Thomson.
Ronald Douglas Thomson.
David Thornton, B.A., B.A.I. (*Dublin*).
Jeevarathinam Victor Thurairasasingam.
Leonard James Tobin.
Hugh Tottenham, B.A. (*Cantab.*).
Kenneth Wilton Trembath, B.E. (*New Zealand*).
Frank Royle Turner.
Louis James Vitali, B.Sc. (Eng.) (*Lond.*).
Robert Ian Walker.
James Ward.
Martin John Peter Warham, B.Sc. (Eng.) (*Lond.*).
Colin Arthur Warner, B.Sc. (Eng.) (*Lond.*).
Anthony William Waterfield.
Michael James Weston.
John Frederick Whitehouse.
Clement Jayatissa Wijetunge.
William Ffrangcon Foulkes Williams.
John David Willson.
Michael Seton Angus Wilson, B.A. (*Cantab.*).
John Leonard Winson.
Kenneth Wright.
Robert Henry Francis Paul Wykes.

The Scrutineers reported that the following had been duly elected as

Members

Ove Nyquist Arup, B.Sc. (*Copenhagen*).
Henry Gustavus Furphy, B.A., B.C.E. (*Melbourne*).
Major-General Laurence Douglas Grand, C.B., C.I.E., C.B.E.
Otto Holden, B.A.Sc. (*Toronto*).
Major-General Henry Porter Wolseley Hutson, C.B., D.S.O., O.B.E., M.C.
Jorgen Saxild.
Charles William Oakey Turner, M.S. (Eng.) (*Illinois*). B.Sc. (Eng.) (*Lond.*).

Associate Members

Peter James Adams, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Stanley Clarke Agnew, B.Sc. (*Belfast*), Stud. I.C.E.
Kenneth Edwin Ainscow, B.Sc.Tech. (*Manchester*), Stud. I.C.E.
Robert Edwin Aitken, B.Sc. (*St. Andrews*), Stud. I.C.E.
Reevel Alderson, B.Sc. (*Durham*), Stud. I.C.E.
Aleck Robinson Aldren, B.Sc. (*Manchester*), Stud. I.C.E.
John Willis Alexander, M.A. (*Cantab.*).
Ratnam Alexander.
John Scott Allen, Stud. I.C.E.
Robert McNair Allen.
Kenneth Guy Anderson.
Conrad Ivar Penton Andren, B.Sc. (Eng.) (*Lond.*).
John Warwick Andrew, B.Sc. (*Leeds*), Stud. I.C.E.
John Frederick Collett Andrews, B.A. (*Cantab.*).
Henry Appel.
William Appleyard, B.Sc. (Eng.) (*Lond.*).
Henry Douglas Armstrong, B.Sc. (*Belfast*).
James Hodgson Armstrong, B.Sc. (*Glas.*), Stud. I.C.E.
Flt.-Lieut. Ernest Ronald Wheeldon Aris.
Maung Aye, B.Sc.Tech. (*Manchester*).
Colin Ayers, B.Sc. (Eng.) (*Lond.*).
Richard Scott Aylward, B.A. (*Cantab.*).
David Andrew Bage, B.Sc. (Eng.) (*Lond.*).
George Ivor Baldwin, B.Eng. (*Sheffield*), Stud. I.C.E.
George Ball.
Michael John Barclay, B.A. (*Cantab.*), Stud. I.C.E.
Brian Dearden Barnes, M.A. (*Cantab.*).
Roy Antony Baxter.
David Drummond Bayley, B.A. (*Cantab.*), Stud. I.C.E.
Arthur Lawrence Hornibrook Baylis, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Peter William Beaumont.
William Collins Bell.
David Birrell, B.Sc. (*Glas.*).
John Black, B.Sc. (Eng.) (*Lond.*).
Stuart Blackford, M.A. (*Cantab.*), Stud. I.C.E.
Richard Sharpe Bolton, B.E. (*New Zealand*), Stud. I.C.E.
Henryk Romuald Borkowski, B.Sc. (Eng.) (*Lond.*).
Alexander Hughes Bowman, B.Sc. (*Glas.*).
Claude Dunne Boyle, B.Sc. (*Glas.*).
Arthur Brebner, B.Sc. (*Aberdeen*).
Harold Pare Briggs, Stud. I.C.E.
Walter Harry Brocklehurst, B.Sc.Tech. (*Manchester*), Stud. I.C.E.
John Brodrick.
Walter Brown.
Ronald Reith Bruce, B.Sc. (*Aberdeen*).
Peter Eugene Bruell, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Tom Roderick Button, B.Sc. (Eng.) (*Lond.*).
Paul Caldwell, D.F.C., B.Sc.Tech. (*Manchester*).
Robert Alexander Callander, B.E. (*New Zealand*), Stud. I.C.E.
James McLean Cameron, Stud. I.C.E.
Joseph Macleod Campbell, B.Sc.Tech. (*Manchester*), Stud. I.C.E.
Richard Frederick Chadney, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Hubert Chadwick.
James Alexander Clark, Stud. I.C.E.
Stanley Clarke, B.Sc. (*Belfast*), Stud. I.C.E.
Alan Clegg, B.Sc. (Eng.) (*Lond.*).
David Norman Coates, B.A. (*Cantab.*).
Frederick William Coates, B.A. (*Cantab.*), Stud. I.C.E.
George Douglas Cochrane, M.A. (*Cantab.*).
Brian Thomas Collins, Stud. I.C.E.
Clarence Maxwell Cone.

JOHN CONSTABLE.
CYRIL WILLIAM COOK, B.Sc. (*Edin.*), Stud. I.C.E.
JOHN NAPIER COOPER, M.A. (*Cantab.*), Stud. I.C.E.
CHARLES ERIC COPELAND.
GORDON EDISON CORRY.
BRIAN COX, B.Sc.Tech. (*Manchester*), Stud. I.C.E.
JOHN RAYMOND CRABTREE, M.A. (*Cantab.*), Stud. I.C.E.
WILLIAM CRANSTON, B.Sc. (*Glas.*).
JOHN ANDREW CROOKS.
IAN KEIR CROSS, B.Sc. (*Glas.*).
ALAN JOSEPH CRYER, B.Sc. (*Leeds*), Stud. I.C.E.
MARCUS LIONEL CURRIE, B.Sc. (*Wales*).
RONALD HERBERT DANIELS, B.Sc. (*Birmingham*).
CHARLES CLARK DAVIDSON, Stud. I.C.E.
ROGER WILLIAM DAVIES.
ALWYN COULSON DAVISON, B.Sc. (*Durham*).
ERIC DAVISON, B.Sc. (*Manchester*), Stud. I.C.E.
DERRICK RICHARD DAY, B.Sc. (Eng.) (*Lond.*).
RONALD WILLIAM HENRY DAY, B.Sc. (Eng.) (*Lond.*).
FRANCIS JOSEPH DEAN, M.Eng. (*Liverpool*), Stud. I.C.E.
CHARLES COMRIE DEWHURST, B.Sc. (*Durham*), Stud. I.C.E.
DONALD CHARLES CASSELS DIXON, B.A. (*Cantab.*).
WALTER HAROLD JAMES DOBSON, B.Sc. (*Bristol*), Stud. I.C.E.
JOHN VIVIAN DRAKE, Stud. I.C.E.
EVELYN FRANCIS DRAPER, B.A., B.A.I. (*Dublin*).
ROY DUNN, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
KENNETH MAYNARD DUNSCOMBE.
CHARLES EMILIEN JULIEN DUPENOIS, B.Sc. (*Birmingham*).
DAVID HENRY DUVALL, B.A. (*Cantab.*).
JOHN NOEL EATHORNE, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
JOHN ANTHONY EDGELEY, Stud. I.C.E.
DOUGLAS HUGH MORLEY EDMUNDS, B.Sc. (*Wales*).
ROBERT WILSON EDWARDS, Stud. I.C.E.
ROBERT EHRLICH, B.Sc. (*Witwatersrand*), Stud. I.C.E.
KEITH GUY EICKHOFF, M.A. (*Cantab.*), Stud. I.C.E.
DEREK LEWIS EVANS.
FREDERICK WILLIAM EVANS.
JOHN WILLIAM EVANS.
FRANK FERGUSON, B.Sc. (*Belfast*).
NISSANKA SRIAPALA SEEMON FERNANDO.
SAMARAPPULIGE WILSON FERNANDO, B.Sc. (Eng.) (*Lond.*).
GERALD JOHN FITZGERALD, B.Sc. (*Glas.*), Stud. I.C.E.
ROBERT PARSONS FLEMING.
PHILIP GEORGE TRUSLER FLINDERS.
MICHAEL SEYMOUR FORINTON, B.A. (*Cantab.*), Stud. I.C.E.
RUSSELL FOX, Stud. I.C.E.
COLIN STUART FRASER, B.E. (*New Zealand*).
MALCOLM DENNIS FREEMAN, B.Eng. (*Sheffield*), Stud. I.C.E.
JAGADISH CHANDRA GANGULY, B.Sc. (Eng.) (*Lond.*), B.Sc. (*Calcutta*).
RONAN LYNAM GARDNER, B.A., B.A.I. (*Dublin*), Stud. I.C.E.
JOZEF GERZON, M.B.E., B.Sc. (Eng.) (*Lond.*).
WARREN ARTHUR GIBSON.
EDWARD RICHARD GILES.
MURDO JOHN GILLIES.
DENNIS RAYMOND GOATLY.
JOHN WALTER GODFREY, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
WILLIAM BAYNE GOODALL, B.Sc. (*Durham*), Stud. I.C.E.
FREDERICK GEORGE GOWDY, B.Sc. (*Belfast*).
DAVID GEORGE GRAINGER, B.A. (*Cantab.*), Stud. I.C.E.
NORMAN SYDNEY JOHN GRASSAM, Ph.D., B.Sc. (Eng.) (*Lond.*).
PETER ROBERT JAMES GRAVES, Stud. I.C.E.
CLEMONS LANDSEER GREEN, B.A. (*New Zealand*).
LESLIE MAURICE GREEN.
GORDON GREENOUGH, B.Eng. (*Liverpool*), Stud. I.C.E.
WILLIAM CHARLES GRIMSEY, Stud. I.C.E.
STANLEY BASNETT GROCOTT.
HEDLEY ROY GUENIN, B.Sc. (*Leeds*).
ARTHUR JOHN CHARLES HADDON, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
JOHN HANSON.
SAMUEL CLIFFORD HARDY, B.Eng. (*Sheffield*).
DERRICK NICHOLS HARRISON, B.Sc. (*Leeds*).
THOMAS MARTIN HART.
WILFRED HARTLEY.
JOHN ALAN HARVEY, B.Sc. (Eng.) (*Lond.*).
THOMAS HAWKES.
ALAN HAYNES, M.A. (*Cantab.*), Stud. I.C.E.
FREDERICK JOHN HEASMAN.
JOSEPH HEATON.
ERIC WILLIAM HEINE, B.E. (*New Zealand*).
HYMAN HENKIN.

Edward John Wilson Henry, Stud. I.C.E.
Reginald Francis Heynes, B.Sc. (Eng.) (*Lond.*).
Anthony Francis Higgins, B.E. (*National*).
Godfrey Hill.
Allan Hodgkinson, M.Eng. (*Liverpool*).
Peter Charles Holderness, B.Sc. (Eng.) (*Lond.*).
John Rosson Hudson, M.B.E., B.Sc. (Eng.) (*Lond.*).
Gwilym Thomas John Hughes.
Thomas David James Hughes, Stud. I.C.E.
John Hunter, B.Sc. (*Glas.*).
Lawrence John Huss, B.Sc. (*Glas.*), Stud. I.C.E.
Kenneth Cobban Imrie, M.Sc. (*Manchester*).
William Imrie.
Fergus Isherwood.
James Harvey Trevithick Ivory, M.A. (*Cantab.*), Stud. I.C.E.
James Allan Jackson, Stud. I.C.E.
Spencer George Jackson, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Thomas Jackson.
John William Andrew Peredur James.
Richard Philip Boyd James, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Frederick William Jenkins, B.Sc. (*Wales*).
Peter Arthur Jessop.
Cledwyn Owen Jones, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Edward Owen Jones.
Stanley Jones, B.Sc. (*Wales*).
John Clark Judson, B.Sc. (Eng.) (*Lond.*).
Chabapathy Kalidasan, B.Sc. (Eng.) (*Lond.*).
David Richard McLean Keil, B.Sc. (*Bristol*), Stud. I.C.E.
Vernon Noel Kelly.
Peter Forbes Kemp.
Robin Mackie Kerr, B.Sc. (*Edin.*), Stud. I.C.E.
Kenneth Arthur Kershaw, B.Sc. (Eng.) (*Lond.*).
Mohammad Khalaf Said Khalaf, B.Sc. (*Fouad*).
Berry Cameron Kidd, M.Sc. (*Belfast*), Stud. I.C.E.
John Henry Garrood King.
Robin Kingston, Stud. I.C.E.
James David Kirby.
Ronald Herbert Knight.
Christopher Gabriel Konarski, Stud. I.C.E.
Lam Po-Hon, M.Sc. (*Lond.*), B.Sc. (*Hong Kong*).
James Crozier Lambie.
Raymond Edward Langley, B.Sc. (Eng.) (*Lond.*).
John Kenneth Lee, B.Sc. (*Manchester*), Stud. I.C.E.
John Lerry, B.Sc. (*Birmingham*).
James Royston Levitt, Stud. I.C.E.
George Colin Irvine Lilly.
John Tony Lineham, Stud. I.C.E.
Henry Russell Lloyd.
Ronald Robert Lloyd, M.A. (*Cantab.*).
Robert Alwyn Lloyd-Owen, B.Sc. (Eng.) (*Lond.*).
Bryan Gilbert Long, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Douglas Carruthers McCallum.
John Arnold Macdonald, B.E. (*New Zealand*), Stud. I.C.E.
John Cameron Forbes Macdonald, B.Sc. (Eng.) (*Lond.*).
Robert McEwan, B.Sc. (*Glas.*), Stud. I.C.E.
Alan Roderick McKay, Stud. I.C.E.
James McNair.
John Jamieson Maconochie.
David Murray MacPherson, B.Sc. (*Glas.*), Stud. I.C.E.
Robert Stewart Livingstone MacVicar.
Peter Gregory Mahoney, Stud. I.C.E.
William Arthur Markham, B.Sc. (Eng.) (*Lond.*).
Clifford Marsden, B.Sc. (*Manchester*), Stud. I.C.E.
Cyril Joseph Mazzetti.
Norman Harold Menesse.
Arthur Henry Mercer, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Peter Merritt, Stud. I.C.E.
Robert Cook Michie, B.Sc. (*Aberdeen*).
James Duke Milne, Stud. I.C.E.
John Black Morrison.
Malcolm Campbell Morrison, B.Sc. (*St. Andrews*).
Norman Morrow, B.Sc. (*Belfast*).
Arthur Morton, B.Sc. (*Manchester*).
Vincent Leslie Mullowney, B.Sc. (*Belfast*).
Dennis Charles Musgrave.
Erik Neustein, B.Sc. (*Edin.*), Stud. I.C.E.
John Willoughby Neve.
Harold Newton, M.Eng. (*Liverpool*), Stud. I.C.E.
Peter Harold Newton.
Norman John Nicholls, B.Sc. (Eng.) (*Lond.*).
Geoffrey Clement Howard Osborne, B.Sc. (*Birmingham*).
Hubert Ovens.
Charles Barry Paget.

Derek Richard Parkinson, B.Sc.Tech. (*Manchester*), Stud. I.C.E.
Martin George Paterson, B.Sc. (*Bristol*), Stud. I.C.E.
Edmond John Patterson, B.Eng. (*Liverpool*).
Edward Clarence Pay, Stud. I.C.E.
Cecil William James Pearson, Stud. I.C.E.
George Herbert Parker Pearson.
Giovanni Fortunato Pesenti, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
John Harry Phillips, Stud. I.C.E.
Edward Daniels Pigott, B.A., B.A.I. (*Dublin*).
Robert Harry Pill.
Leslie Ponsford, B.Sc. (*Wales*).
Kenneth Oswald Pook, B.Sc. (Eng.) (*Lond.*).
Derrick Postlethwaite, Stud. I.C.E.
George Christopher Poston, B.Sc. (*Edin.*).
Ian Martin Dalgleish Potter, M.A. (*Cantab.*).
William Gordon Powell.
Alfred Glenton Preston, M.Sc. (*Durham*).
David Richard Prestwich, B.A. (*Cantab.*).
Ernest Lilburn Pye, M.Eng. (*Liverpool*), Stud. I.C.E.
Llewelyn James Redman.
Eric Alfred Reed, Stud. I.C.E.
Kenneth Reynolds.
Norman Johnson Reynolds.
Lawrence Kenneth William Richards, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Barrie Ewart Philip Edward Roberts, B.Sc. (*Wales*).
Douglas Gordon Robertson, M.Eng. (*Liverpool*).
Douglas Hicks Robertson, Stud. I.C.E.
George Simpson Rollo.
Sydney Rosen.
Maurice James Rowbottom, B.Sc. (*Birmingham*), Stud. I.C.E.
Arthur Percy Russell, Stud. I.C.E.
Peter Rufus Sanderson, B.Sc. (*Manchester*), Stud. I.C.E.
John Ernest Francis Sandford.
Veluppillay Santhirasenan.
Albert Reginald Sargent.
John Rienzie Noel Scharenguivel.
Alan James Scott, B.Sc. (*Durham*), Stud. I.C.E.
Knud Gyldenstierne Sehested.
Ralph Southern Sharrock, M.Eng. (*Liverpool*).
Charles Arnold Simmonds, M.A. (*Cantab.*), Stud. I.C.E.
Roy Douglas Sissons.
William Edward Albert Skinner, Stud. I.C.E.
Ronald James Slater, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
George Rintoul Smail, M.A. (*Edin.*), B.Sc. (*Edin.*).
John Malfaison Smales, B.Sc. (Eng.) (*Lond.*).
Brian Groome Smith, B.Sc. (Eng.) (*Lond.*).
Bryan Leonard Chesterton Smith.
Frank Millward Smith.
James Crichton Renton Smith, Stud. I.C.E.
Michael John Chambers Smith, Stud. I.C.E.
Stanley William Smith.
Norman Sneath, B.A. (*Cantab.*), Stud. I.C.E.
David Bruce Snushall, M.A. (*Cantab.*), Stud. I.C.E.
Robert Soutar.
Arthur Maurice Sowden, M.A. (*Oxon*).
Peter Henry Steel, B.Sc. (*Bristol*), Stud. I.C.E.
Gordon Stevenson.
Graham Lindsey Steventon, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
George Walker Stewart.
Howard MacMillan Stewart, B.Sc. (Eng.) (*Lond.*).
Wilfred Basil Stilwell, B.E. (*New Zealand*).
Robert John Cameron Stobie, B.Sc. (*St. Andrews*), Stud. I.C.E.
Cyril Gardner Strange, B.Sc. (Eng.) (*Lond.*).
Vane Sutton-Vane.
Ronald Sykes.
Geoffrey Leonard Symes.
James Symons.
Robert Taylor, Jnr., B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
William Taylor, Stud. I.C.E.
Leonard John Teasdale.
John Henry Tempest, Stud. I.C.E.
William Niven Thom, Stud. I.C.E.
Ian Maclachlan Thompson.
Eric Bone Thomson, B.Sc. (*Glas.*).
Frank William Thomson, B.Sc. (*Manchester*), Stud. I.C.E.
George Tidy, M.A. (*Cantab.*).
Arthur Bray Tiffen, B.E. (*New Zealand*).
Abner John Charles Tigg.
John Tiplady, B.Sc. (*Leeds*).
Magnus William Todd, B.Sc. (*Edin.*).
Geoffrey Lovell Tomkins, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Geoffrey Arthur Toynbee, B.E. (*New Zealand*), Stud. I.C.E.
Cecil Henry Trent, Stud. I.C.E.

William Graham Howard Tripp, B.A. (*Cantab.*).
Bertram Eric Twyford, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Alan Varty, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Dennis Tracy Vaulkhard.
Charles Dennis Harling Vernon, M.A. (*Cantab.*).
Robert Harry Vincent, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Roman Ludwik Wajda.
Kenneth Derek Wale, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
George Philip Erroll Walker, B.Sc. (Eng.) (*Lond.*).
Peter McKay Walker, B.Sc. (*Manchester*), Stud. I.C.E.
William Wallace, B.Sc. (*Belfast*), Stud. I.C.E.
John Arthur Walters, B.Sc. (Eng.) (*Lond.*), Stud. I.C.E.
Anthony Herbert Ward, M.A. (*Cantab.*), Stud. I.C.E.
Denis Mervyn Watkins, B.Sc. (*Birmingham*).
William James Graham Watson, B.E. (*New Zealand*).
John Douglas Watts, B.Eng. (*Liverpool*), Stud. I.C.E.
Charles Henry Webb, Stud. I.C.E.
Harry Eric Welbourn.
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Paper No. 5834

"A Laboratory Investigation of Some Bridge-Deck Systems" * †

by

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Synopsis

The Paper describes an investigation, undertaken at the request of the Ministry of Transport, of the behaviour of some bridge-deck systems of steel-and-concrete construction when subjected to concentrated loading. The theoretical analysis is only briefly described but the main results of the experimental work, involving the testing of four model bridge decks in the laboratory, are presented and discussed.

The present methods of design for such bridges are considered and possible modifications put forward. Alternative design methods are suggested.

Introduction

In a composite structure, the aim of the designer is to combine different materials so that they interact efficiently, whereby the stiffness and strength of the structure are considerably greater than would be possible if the materials were arranged to act independently. Often, the choice of materials and their relative disposition are based on the mutual correction of the structural deficiencies of the separate materials; reinforced concrete is the most familiar form of such construction in modern times.

In present-day engineering design, the term "composite construction" is particularly adopted for systems in which rigid load-bearing steel beams or stanchions are embedded in, or otherwise attached to, concrete. In buildings, a form of this construction is the filler-joist floor; in the design of such a floor, the engineer may calculate the strength on the basis of full structural interaction between steel and concrete, or he may base his computations on the carrying capacity of the beams alone, introducing an increase in stress to allow in some measure for the strengthening effect of the

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concrete. In multi-storeyed buildings of steel-frame construction, it is common to encase the frames in concrete to give greater resistance to fire. In the past, the effect of this concrete in stiffening and strengthening the steelwork has been largely disregarded, but in recent Codes of Practice some allowance for the effect has been introduced, leading to more economical design.

In bridges, too, composite structural forms have been used in the past, without taking account of composite action in design, or merely regarding it as an additional reserve of strength. Cast-iron girder bridges, for example, have been shown to have a stiffness and strength vastly exceeding the estimates of their designers, when the bridges were tested some 70 years after erection. Steel-and-concrete bridges have become increasingly popular in the last 20 years, particularly for short spans. In this latter connexion, it is interesting to note that of the 3,600 bridges (of all types) built, reconstructed, or widened in Great Britain in the 6 years preceding the last war, 84 per cent were less than 50 feet in span, 11 per cent had spans between 50 and 100 feet, and only 5 per cent had spans longer than 100 feet.[1]

A considerable amount of research has been made abroad on various aspects of composite construction, both experimentally and analytically. Work at Illinois University [2] has dealt with the slab-and-girder type of bridge deck, for both square and skew spans, simply supported and continuous. Investigation of the different types of shear connectors, for developing structural interaction between the steel beams and the concrete, has been made at Illinois, at the Federal Institute for Testing Materials at Zurich, and, more recently, at Stuttgart.[3]

This Paper describes work at the Building Research Station, undertaken at the request of the Ministry of Transport, on the behaviour of different types of composite bridge-deck slabs, when subjected to concentrated loads, both for working conditions and also at failure. Alternative proposals for the application of the results to practical design are tentatively made, in the hope that the reactions of practising engineers to these proposals may enable the future research programme to be directed into the most useful paths.

Types of Composite Bridge-Deck Systems

Cast-Iron Girder Bridges

There are about 4,000 cast-iron girder bridges in the British Isles. In these bridges, the girders, spaced up to 7 feet apart, were originally assumed to support the whole dead and live load, without any allowance for the strengthening due to composite action between the girders, the brick jack-arches spanning between them, and the top fill. It is, in fact, probable that such composite action was of little significance immediately after the

[1] The references are given on p. 172.

construction of a bridge ; but in the course of time the compaction of the fill led to the development of a composite section in which the cast iron provides the tension zone, and the brick arches, combined with the fill, act generally in compression.

About one-half of the total number of such bridges were originally classified as unfit to carry loads exceeding 12 tons. Tests made by the Building Research Station in co-operation with the Ministry of Transport resulted, however, in the acceptance of a more accurate assessment of the strength of cast-iron girder bridges.[4] It was concluded from these tests that the live-load stresses should be estimated on the basis that composite action effectively increases the section modulus of the girders by a factor D/d, where D denotes the overall depth of the bridge section and d the depth of the girder.

Since it is unlikely that cast-iron will be used in the construction of new bridges, the aim of the investigation was merely to provide a means of obtaining a fair estimate of the safe load-bearing capacity of existing bridges. For steel-concrete bridges, however, it is necessary to develop, for new bridges, a rational design method which is reasonably accurate. The method must not be unduly complicated but is unlikely to be capable of such extreme simplification as that adopted for checking the strength of existing cast-iron girder bridges.

Composite Steel-Concrete Bridges

The use of concrete and reinforced concrete in conjunction with steel girders in bridges, lends itself to the development of a variety of structural combinations. In the main, the variations appear in the degree of encasement of the girders in concrete, and in the method used to ensure adequate bond at the steel-concrete interface.

The simplest form of composite steel-concrete construction is the filler-joist slab, where steel girders are entirely encased in concrete. The distance between adjacent beams does not generally exceed 2 feet 6 inches and steel-mesh reinforcement is usually provided at the top and bottom of the slab. For reasonable economy, bridges of filler-joist construction do not often exceed 20 to 25 feet span ; however, such bridges with spans of 35 feet and more have been built, where headroom limitations made this type of construction desirable.

Jack-arch construction is commonly used for spans up to 40 feet, but has been adopted, exceptionally, up to 65 feet. The distance between adjacent girders does not usually exceed about 5 feet. The bottom flanges of the girders are often encased in concrete before erection, forming skew backs from which the concrete jack-arches spring. Tie-bars are normally provided in the edge panels and sometimes elsewhere, to help in resisting the horizontal thrust developed by the arches. In some bridges, tie-bars are omitted, and the edge panels are specially strengthened to resist the thrust. The arches are often cast on a permanent shuttering supported

on the bottom flanges of the girders. In the jack-arch system the stresses in the concrete, both parallel to the span and transversely, are largely compressive and reinforcement is therefore unnecessary.

If steel reinforcement is included in the slab, particularly at right angles to the girders, a part of the concrete at the springing of the arches may be omitted. The bridge then consists of a reinforced-concrete slab supported on encased girders, a structure similar to the conventional reinforced-concrete slab-and-girder floor. If the girders are not encased in concrete, the system loses much of its composite character unless special shear connectors are used to tie the slab to the girders.

In steel-concrete bridges of the type discussed above, the distance between the girders varies usually between one-third and one-seventh of the span, which rarely exceeds 70 feet. For very large spans, secondary or tertiary beams are used to support the slab.

The construction of these bridges is simple and does not require much skilled labour. The girders can be used to support the shuttering for the concrete and construction can proceed without serious interference with traffic beneath the bridge.

A composite section of two different materials can be effectively formed only if the shear forces at the surfaces of contact, between the two elements, are transmitted without relative displacement. In filler-joist and jack-arch slabs, where the steel is fully encased, the bond between the girders and the concrete is sufficient for this purpose. In slab-and-girder bridges, however, the bond between the concrete and the top flange of the beam cannot be relied upon to maintain efficient shear transmission, and special anchorages must be incorporated to ensure composite action.[3]

Theoretical Investigation

The mathematical analysis of the stress distribution in bridge-deck systems of the type considered in this Paper offers considerable difficulty because of the complex mechanical properties of the materials and also because of the number of parameters that need to be taken into account. Two limiting systems have been chosen for theoretical investigation : first, an idealized slab-and-girder system and, second, an idealized filler-joist system.

Slab-and-Girder System

The behaviour of the slab-and-girder system has been studied for the following simplified conditions :

(1) the materials are considered to be completely elastic ;
(2) a number of equally spaced parallel girders at right angles to the abutments support a slab of uniform flexural rigidity ;
(3) the slab and girders are simply supported at the abutments, that is to say, the deflexions and bending moments are zero ;

(4) the girders are of uniform stiffness throughout;
(5) the loading consists of a point load normal to the slab;
(6) the girders offer no torsional restraint;
(7) the girders and slab always maintain contact throughout the length of the girders even when the slab tends to lift, but there is no transmission of shear between the two materials.

The initial calculations were concerned with the bending moments at mid-span for point loads at various positions along the mid-span line.

By giving the structure the above properties, the number of parameters is reduced to only two, besides the position of the load. They are (*a*) the ratio of girder stiffness to slab stiffness and (*b*) the spacing of the girders. For convenience, they are stated in non-dimensional terms as follows:—

$$\text{Stiffness ratio,} \quad K = \frac{\pi E_s I_g}{4Dl} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Spacing coefficient,} \quad b = \frac{\pi b'}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where E_s denotes Young's modulus for the steel girder;
I_g ,, the moment of inertia of the girder section;
D ,, the flexural rigidity of the slab, that is to say,

$$D = \frac{E_c h^3}{12(1-\nu^2)};$$

l ,, the length of span;
E_c ,, Young's modulus for the concrete slab;
h ,, the thickness of the slab;
ν is Poisson's ratio for the material of the slab; and
b' denotes the spacing of the beams.

By making all the calculations in terms of non-dimensional quantities, the bending moments are obtained in the form of coefficients which are then multiplied by the appropriate dimensional factors.

Even after all these simplifications, the amount of computation for all possible widths of bridge would be prohibitive and the resulting tables of results too unwieldy for practical use. To examine the effects of K and b on the bending moments in the girders, it was decided to consider first a system in which there is an infinite number of girders so that the distribution of bending moments is the same whatever girder is loaded. It was found that the maximum bending moment in the girders increased with both K and b.

The effect of introducing a free edge into this system was then examined, the resulting arrangement being referred to as a semi-infinite system. An example of the results obtained are given in Table 1, for particular values of the parameters, $K = 1{\cdot}0$ and $b = 0{\cdot}5$. The girders are numbered G1 (edge girder), G2, G3, etc., and the bending moment coefficients μ are tabulated for a point load applied over each girder, at mid-span, in

turn. These coefficients can be converted into actual bending moments at mid-span by the relationship :

$$M = \frac{2Pl}{\pi^2} \cdot \mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where P denotes the value of the point load.

Thus, for a load over the edge girder, the bending moment resisted by that girder is $2 \times 0{\cdot}735Pl/\pi^2 = 0{\cdot}149Pl$ as compared with $0{\cdot}25Pl$ if the beam alone carried the load.

TABLE 1.—COEFFICIENTS ($= 1{,}000\mu$) FOR CALCULATING THE BENDING MOMENTS IN THE GIRDERS OF A SEMI-INFINITE SYSTEM (WITH $K = 1{\cdot}0$, $b = 0{\cdot}5$)

Girder for which coefficient is given	Girder over which the load is placed at mid-span :									
	G1 (edge)	G2	G3	G4	G5	G6	G7	G8	G9	G10
G1	735	290	112	32	−1	0	−2	−3	−2	0
G2	290	467	215	101	40	12	0	−2	−3	−2
G3	112	215	428	204	102	41	12	0	−2	−3
G4	31	102	204	420	200	100	41	12	0	−2
G5	−2	41	101	200	417	197	100	41	12	0
G6	−9	10	42	99	198	415	197	100	41	12
G7	−7	−1	12	42	100	198	415	197	100	41
G8	−4	−3	0	12	41	100	198	415	198	101
G9	−1	−3	−3	0	12	41	100	198	415	197
G10	−1	−1	−3	−2	0	12	41	100	198	415
G11	0	−1	−1	−3	−2	0	12	41	100	197
G12		0	−1	−1	−3	−2	0	12	41	101
G13			0	−1	−2	−3	−2	0	12	41
G14				0	0	−2	−3	−2	0	12
G15						0	−2	−3	−2	0
G16							0	−2	−3	−2
G17								0	−2	−3
G18									0	−2
G19										0

From Table 1 it will be seen that, when the load is placed over girder G6 or any girder further from the free edge, the system behaves as an infinite slab with an error of less than one per cent of the maximum value of μ. A useful simplification of the results is thus possible by considering that the bending moment for any girder in a semi-infinite system, is the sum of the value corresponding to an infinite system together with a correction depending on the distance of the girder from the edge. The correction for the case previously considered is shown in Table 2.

For a bridge of finite width, the results for the semi-infinite system can often be used with sufficient accuracy by introducing the corrections, as

TABLE 2.—CORRECTIONS ($= 1{,}000\ \Delta\mu$) TO BE APPLIED TO THE BENDING-MOMENT COEFFICIENTS ($= 1{,}000\ \mu$) CORRESPONDING TO AN INFINITE SYSTEM, TO ALLOW FOR THE EFFECT OF AN EDGE

Girder for which the correction is given	Girder over which the load is placed at mid-span :				
	G1 (edge)	G2	G3	G4	G5
G1	320	93	11	−9	−11
G2	93	52	18	0	−1
G3	11	18	13	7	1
G4	−10	1	7	5	3
G5	−12	0	0	3	2
G6	−9	−2	1	−2	1
G7	−5	−1	0	1	−1
G8	−1	−1	0	0	0
G9	1	0	−1	0	0
G10		1	0	0	0
G11			1	0	0
G12			−1	1	0
G13				−1	0

given in Table 2, to both edges. For example, for a system of six girders with the load over one of the middle girders, the computations are shown in Table 3. The bending-moment coefficients are first set out for the middle eleven girders of an infinite system and successive corrections are then

TABLE 3.—AN EXAMPLE OF THE METHOD OF DERIVING THE BENDING-MOMENT COEFFICIENTS ($= 1{,}000\ \mu$) FOR THE GIRDERS OF A FINITE SYSTEM

Girder for which coefficients are given	G1 (edge)	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11
Bending-moment coefficients for infinite slab (load over G6 at mid-span).	0	12	41	101	197	415	197	101	41	12	0
Corrections for edge at G3.			−9	0	7	5	3	−2	1	0	0
Corrections for edge at G8.	0	0	1	0	7	13	18	11			
Bending-moment coefficients for finite slab.			33	101	211	433	218	110			

made for the edges of the finite system, formed at girders G3 and G8. It is evident from the order of the corrections outside the range of the finite bridge that no error is involved in this simple transformation from the semi-infinite to the finite case, for the example considered.

Filler-Joist System

The analysis of the deformation of filler-joist slabs under the action of concentrated loads was made on the primary assumption that the slab could be considered equivalent to a thin plate of a homogeneous anisotropic material, having differing elastic properties in the directions of its three perpendicular axes of symmetry. The usual assumptions of " thin-plate " theory were made and the problem was treated in terms of plane stress. This inevitably involves some distortion of the stress pattern particularly in the vicinity of the concentrated load where the transverse stress acting through the plate is not negligible, but this can be allowed for in the analysis by using a larger area of application of the load than is actually the case.

The differential equation governing the deflexion w of such an anisotropic plate under a normal load q (x,y) is :—

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q(x,y) \quad . \quad . \quad . \quad . \quad (4)$$

for which the equivalent elastic constants D_x, D_y, and H may be readily evaluated in terms of the properties of the given filler-joist system.

Equation (4) was solved for the case of a slab simply supported on two opposite sides and free on the other two sides. Much of the numerical work was concerned with the maximum bending moment under the load ; the effects of width-to-span ratio and the relative stiffness of the slab in the directions parallel and perpendicular to the span were investigated.

The simplification of the results for practical use over a wide range of practical conditions involves considerable difficulty but it seems from the work so far completed that such simplification may be possible without important loss of accuracy. It has been found that the bending moments are largely controlled by the values of two parameters :

first, α where

$$\alpha = \sqrt{\tfrac{1}{2}\left(\frac{H}{D_y} + \sqrt{\frac{D_x}{D_y}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

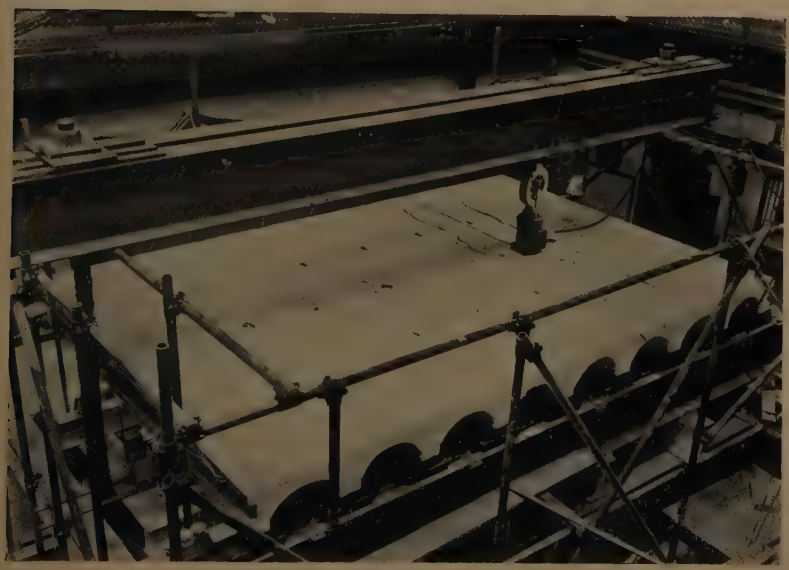
and secondly, the width/span ratio.

For the usual practical range of designs, other factors are relatively unimportant ; and a graphical presentation of the results suitable for designers should readily follow the completion of the lengthy computations based on the fundamental theory.

Fig. 2

MODEL S.G.(S) (SLAB-AND-GIRDER SYSTEM WITH SHEAR CONNECTORS). VIEW OF GIRDERS AND SHUTTERING

Fig. 3



MODEL J.A. (JACK-ARCH SYSTEM) AFTER FAILURE UNDER A LOAD PLACED OVER A GIRDER AT MID-SPAN

Fig 4

MODEL F.J. (FILLER-JOIST SLAB)

Experimental Investigation

General

Tests on full-scale bridges showed that the stresses and deformations due to normal traffic, or even to special test-vehicles, were too small to enable a satisfactory study to be made of the general stress distribution throughout the bridge. Moreover, comprehensive tests were impractical owing to limitations to the time for which a bridge could be occupied by a loading vehicle to the exclusion of other traffic. It was therefore necessary to make tests on model bridge slabs, large enough to give reasonable correspondence with the full-scale bridge yet sufficiently small for the tests to be made in a laboratory under controlled conditions. From these considerations the linear scale adopted for the main tests was one-third except for the filler-joist slab for which the scale was one-half. For each type of bridge deck, a simplified typical cross-section of an actual bridge was prepared by the Ministry of Transport.

The results of tests on four slabs are discussed in this Paper. These were :—

(1) Model S.G. A slab-and-girder system, without shear-connectors.
(2) Model S.G.(s). A slab-and-girder system, with shear-connectors.
(3) Model J.A. A jack-arch system.
(4) Model F.J. A filler-joist system.

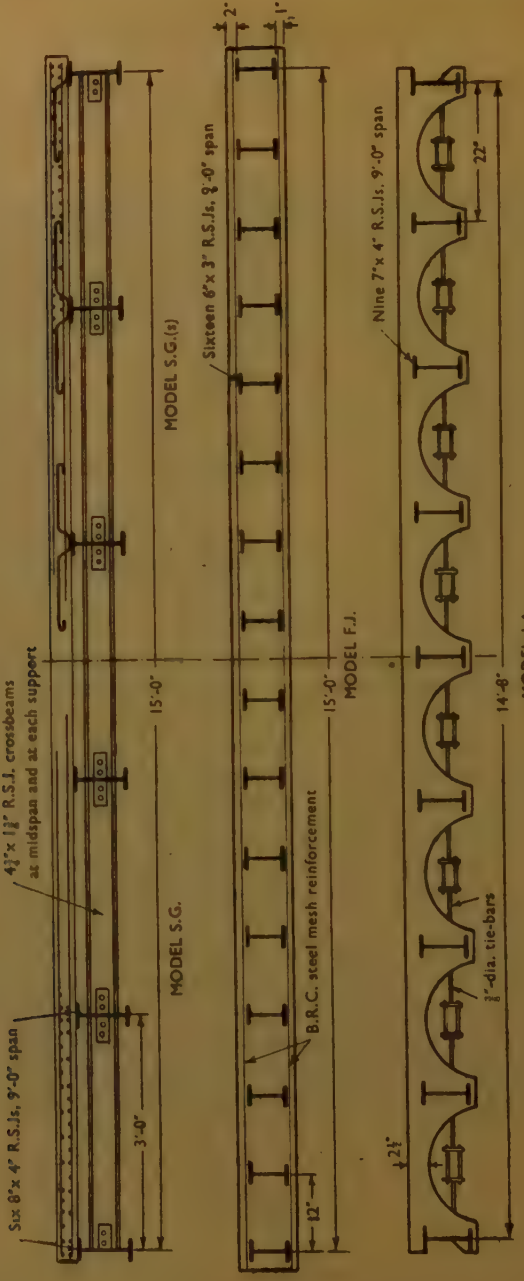
In addition to these slabs, smaller slabs were tested to investigate some secondary effects and the influence of different methods of construction.

The main slabs tested are shown in *Figs 1* to *4*, and Table 4 gives the constructional data relating to each model slab, in conjunction with those for the original, full-size bridge. The properties of the materials used for the models were similar to those used in practice. The concrete was prepared, placed, and cured at a constant temperature of 64° F. and a relative humidity of 65 per cent. Details of the properties of the materials used are given in Table 5. For purposes of calculation, the modular ratio was assumed to be 6.

Dimensional aspects.—According to dimensional theory, a true model, built of the same materials as the original structure and reduced linearly in all directions in the same ratio $1/n$, will develop the same bending stresses as in the full-scale structure if the magnitude of the load is reduced for the model by $1/n^2$ for point loads and $1/n$ for line loads ; for distributed loads the same stresses result for all corresponding models if the intensity of loading (that is to say, load per unit area) is the same.

For the equivalent load which satisfies this stress criterion, the deflexion of the model is $1/n$ times that of the original structure ; thus the configuration of the loaded model is a scale representation of the deformation of the full-size bridge.

Test arrangements.—The slabs were tested in two frames which served to transmit the loads (see *Figs 3* and *4*). Generally, the concentrated load



Figs 1

CROSS-SECTIONS OF MODEL SLABS TESTED (SPAN 9' 0")

Bridge-deck-system	Size	Steel girders						Transverse members			Slab Thickness: in.	Slab reinforcement				
												Normal to beams		Parallel to beams		
		No.	Size of section	Span: ft	Spacing:	Moment of Inertia: in.^{4}	Section modulus: in.^{3}	Size	Spacing	Attachment to beams		Dia.: in.	Spacing: in.	Dia.: in.	Spacing: in.	Cover: in.
Slab-and-girder systems	Full scale	6	Top flange: 12″ × 1⅛″ Bottom flange: 10″ × 1⅛″ Web: 24″ × ⅜″	27	9′ 0″	4,316	308	BSB 130, 15″ × 6″ × 45 lb.	At mid-span and at supports	Welded to web stiffeners	9	¾	5	⅝	6	⅞
	⅛ scale (Models S.G. and S.G.(s))	6	BSB 112, 8″ × 4″ × 18 lb.	9	3′ 0″	55·6	13·9	BSB 105, 4¾″ × 1¾″ × 6·5 lb.	At mid-span and at supports	Bolted through 6″ × 3″ × ⅜″ angle brackets	3	¼	1⅝	3/16	2	½
Jack-arch system	Full scale	9	Top flange: 10″ × ¾″ Bottom flange: 10″ × ¾″ Web: 24″ × ⅜″	27	5′ 6″	2,730	214	—	—	—	8 at crown of arch	—				—
	⅛ scale (Model J.A.)	9	BSB 111, 7″ × 4″ × 16 lb.	9	1′ 10″	39·5	11·3	⅜″ dia. mild-steel tie-bars	At mid-span and at ¼-span	Bolted through webs of beams	2½ at crown of arch	—				—
Filler-joist system	Full scale	16	BSB 125, 13″ × 5″ × 35 lb.	18	2′ 0″	283·5	43·6	—	—	—	18	B.R.C. Fabric No. 8				1
	½ scale (Model F.J.)	16	BSB 108, 6″ × 3″ × 12 lb.	9	1′ 0″	21·0	7·0	—	—	—	9	B.R.C. Fabric No. 14				½

TABLE 5.—PROPERTIES OF THE MATERIALS USED IN THE MODEL SLABS

Model	Steel				Concrete			
	Component	Yield-point stress : Tons per sq. in.	Stress at ultimate failure : Tons per sq. in.	Modulus of elasticity : lb. per sq. in.	Mix (by weight)	Crushing strength* lb. per sq. in.	Transverse strength* : lb. per sq. in.	Modulus of elasticity* : lb. per sq. in.
S.G.	Slab reinforcement : $\frac{1}{4}$" dia. steel bars	20·5	30·0	29·5	1 : 2·27 : 2·73 w/c = 0·575	5,200	580	$4{\cdot}6 \times 10^6$
S.G.(s)	Girders : BSB 112, 8" × 4" × 18 lb.	14·0	26·0	29·5	1 : 2·27 : 2·73 w/c = 0·575	5,100	540	$4{\cdot}8 \times 10^6$
	Slab reinforcement : $\frac{1}{4}$" dia. steel bars	20·5	27·5	29·4				
J.A.	Girders : BSB 111, 7" × 4" × 16 lb.	—	—	29·5	1 : 2·27 : 2·72 w/c = 0·575	4,500	—	$4{\cdot}9 \times 10^6$
F.J.	Girders : BSB 108, 6" × 3" × 12 lb.	—	—	28·5	1 : 2·27 : 2·72 w/c = 0·575	5,000	510	$4{\cdot}4 \times 10^6$
	Slab reinforcement : B.R.C steel mesh	—	39·5	—				

* At 28 days.

was applied, by means of a hydraulic jack, through a 3-inch-diameter steel disk bedded on the concrete slab in plaster of Paris. The load was measured with a ring-type load-gauge. The slabs were freely supported on a span of 9 feet.

The load was applied at mid-span or quarter-span, either immediately over a girder or on a concrete panel mid-way between girders. In the " working-load " tests, the applied load ranged from $1\frac{3}{4}$ ton to $3\frac{1}{2}$ tons depending on the position of the load and the type of slab tested.

Measurements were made, during the tests, of the deflexions at mid-span, quarter-span, and supports ; for this purpose, dial-gauges reading to 0·001 inch or 0·0001 inch were used. Strain measurements were made, with electrical resistance strain-gauges at mid-span and at quarter-span, on the top and bottom flanges of all girders, and also at various positions on the concrete slab or its reinforcement. At points where the strains were largest, Huggenberger tensometers were also used.

The forces in the tie-bars of the jack-arch system were deduced from measurements of the strains in the bars. In the test of slab S.G.(s), resistance strain-gauges were fixed to the shear-connector bars embedded in the concrete.

In the first " working-load " tests, the application of the load produced no visible cracking of the concrete. In order to simulate the conditions that may arise in practice after a considerable period of service, the slab was systematically cracked by placing loads successively at various positions along the middle of each panel of the slab. The " working-load " tests were then repeated. Repetitions of the loading were also necessary to study other alterations in the condition of the slab, for example, the removal of tie-bars in the jack-arch slab or of the transverse diaphragms in the slab-and-girder system.

The slabs were subsequently tested to destruction, first by punching through a concrete panel and then by applying the load directly over a girder until bending failure occurred.

Testing programme.—The investigation was planned primarily to determine the effect of various factors on the behaviour of the slabs, with particular reference to the following aspects :—

(1) Composite action between the girders and the concrete.
(2) The lateral distribution of load amongst the girders.
(3) The ultimate strength and mode of failure for loads at various positions.

The influencing factors considered were :

(*a*) the type of deck slab ;
(*b*) the presence of cracking ;
(*c*) the provision of shear-connectors in a slab-and-girder system ;
(*d*) the cross-beams in a slab-and-girder system and the tie-bars in a jack-arch system ;

(*e*) the scale of the model; and
(*f*) the degree of load concentration.

The experimental work resulted in the accumulation of a considerable quantity of test data. These data could not be presented in their entirety within the confines of the present Paper. Moreover, it was felt that detailed exposition of experimental evidence would be of less interest to engineers than the methods used to obtain it and the possible application of the results to the solution of engineering problems. Only the salient features of the results are therefore collated here. The whole of the information will be included later in a number of Research Papers in the National Building Studies series.

" Working-Load " Tests

Within the range of loading corresponding to practical conditions, the models behaved elastically. Although the value of the applied load was varied somewhat for different tests, the strains and deflexions recorded in this section are presented in terms of the effect of a 10,000-lb. load, to allow easy comparison of the results.

Edge-girders and intermediate girders.—Theoretical consideration indicates that the maximum deformation and stress occur beneath the load when it is placed at mid-span at the free edge of the system, and the strains and deflexions measured in the tests confirmed this. The ratio of the maximum girder strains for loading over the edge girder and over a girder near the centre of the system, respectively, was about $1\frac{3}{8}$ for the slab-and-girder system (with or without shear connectors), $1\frac{3}{4}$ for the jack-arch slab, and $2\frac{1}{2}$ for the filler-joist slab (see Table 6). These results apply to the initial " uncracked " condition of the models, and similar ratios were found for the maximum deflexions, immediately under the position of loading.

Not only was the maximum strain greatest for edge-loading but the sum of the strains at mid-span of each of the girders was also greatest when the load was over the edge girder. This result agrees with that deduced from the theory, that the ratio of the aggregate longitudinal bending moment borne by the girders to the applied bending moment increases as the position of loading approaches the edge.

The results for the filler-joist slab indicate that this slab was behaving similarly to an isotropic plate. In the remaining models, the maximum bending moments in the various girders differed less, owing to the stiffening effect of the girders. It is evident that, if the girders are very stiff compared with the slab, the girder immediately under the load will resist most of the applied bending moment, at whatever position the load is placed relative to the free edge, the distributing function of the slab being small. The ratio of the maximum bending moments for loading over an edge girder and over a girder near the centre of the system, respectively, would therefore approach 1·0. With increasing slab stiffness the ratio would be

TABLE 6.—MAXIMUM STRAINS IN THE GIRDER FLANGES

Position of load	Position of strain-measurement			Strain × 10^6, for a load increment of 10,000 lb.*							
	Girder	Span position	Flange	Model S.G.		Model S.G.(s)		Model J.A.		Model F.J.	
				Uncracked	Cracked	Uncracked	Cracked	Uncracked	Cracked	Uncracked	Cracked
Mid-span of edge girder	Edge girder	Mid-span	Top	−340	−415	+ 15	+ 6	− 90	−155	−75	− 70
			Bottom	+335	+435	+330	+315	+210	+280	+93	+135
		¼-span	Top	−195	−235	− 1	− 3	− 30	− 40	−20	− 20
			Bottom	+195	+240	+ 15	+140	+ 70	+ 90	+15	+ 25
Mid-span of intermediate girder	Intermediate girder	Mid-span	Top	−178	−259	− 40	− 44	− 28	− 36	−33	− 33
			Bottom	+193	+273	+189	+223	+173	+123	+34	+ 54
		¼-span	Top	− 83	−140	—	—	− 6	− 7	− 6	− 8
			Bottom	+107	+146	+ 80	+ 80	+ 31	+ 49	+ 6	+ 10

* *Note:* A positive sign indicates tensile strain, and a negative sign indicates compressive strain.

expected to increase until, in the isotropic slab without girders, the corresponding ratio in terms of maximum bending moments per unit width reaches a value of 2·6.

Cracking of the slab reduces its efficiency in distributing the effects of the loading to parts of the system remote from the point of application of the load, and hence cracking resulted in a reduction of the ratio of maximum bending moments discussed above, by from 6 to 20 per cent for the various models.

Effect of shear connectors.—The effect of shear connectors is indicated by a comparison of the behaviour of models S.G. and S.G.(s) (see *Figs 10* and *11*, and Table 6). The maximum measured flange strains were the same for both models before the commencement of cracking. Although the advantage of shear connectors was thus not apparent in the values of the flange strains, the effect on the stiffness of the models was marked, the provision of the connectors leading to a 50-per-cent reduction in maximum deflexion.

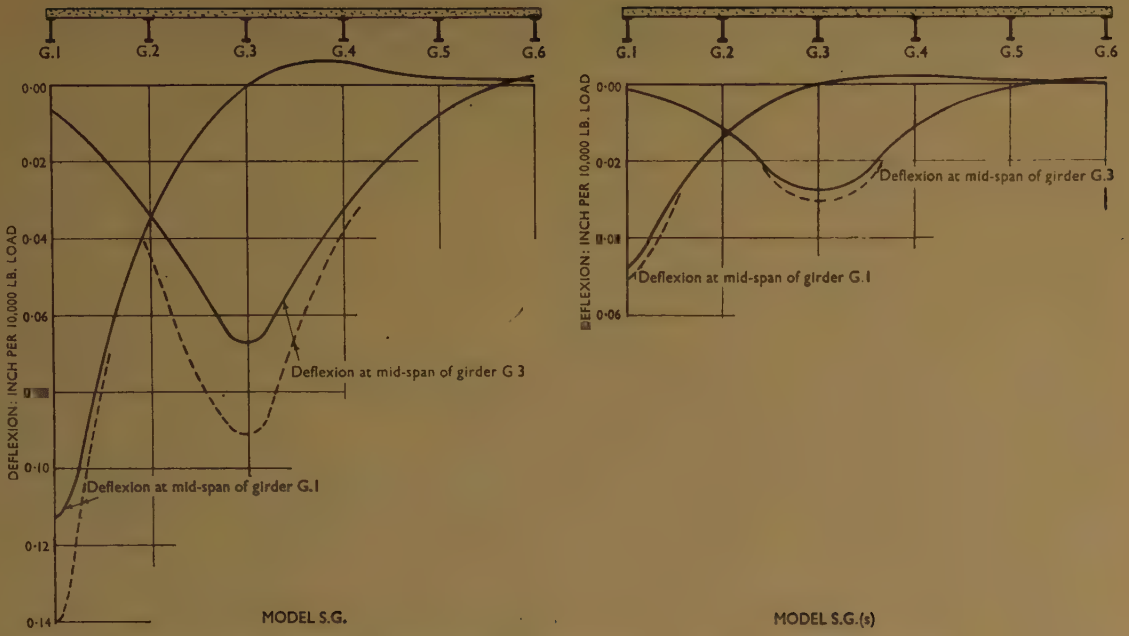
The reason why the inclusion of shear connectors did not result in reduced girder stresses is that the girders were effectively stiffened by composite action with the concrete, so that the division of the total bending moment between the slab and the girders was altered. It appeared from the experimentally determined position of the neutral axis that there was very little composite action in model S.G. but there was full composite action in model S.G.(s). On this basis it was estimated from the measured strains that for loading on an intermediate girder before cracking of the slab, 55–60 per cent of the total applied bending moment was resisted by the beams in model S.G. whereas in model S.G.(s) this proportion was approximately 80 per cent. The proportions obtained from the theoretical investigation were about 80 and 90 per cent, respectively, so that the theory gives an over-estimate of the girder stresses for the system where there is no composite action, but is satisfactory for the stiffer composite-girder system of model S.G.(s).

Whereas the girder strains are dependent on the section modulus of the effective cross-section, the deflexions, on the other hand, depend on the moment of inertia. The latter was increased much more by composite action, owing to the provision of shear connectors, than the section modulus; and the calculated deflexions for model S.G. were about one-half of those for model S.G.(s). This result agrees with that obtained experimentally and recorded above.

After cracking, the advantage of the shear connectors was more obvious. The maximum stresses in model S.G. were 20 to 30 per cent lower than for model S.G.(s), and the deflexion of the model with shear connectors was only about one-third of that of the other model.

It is of interest to note from Table 6 that the maximum flange strains at the quarter-span section are less in model S.G.(s) than in model S.G. even before cracking; so that for combinations of loads, such as arise from

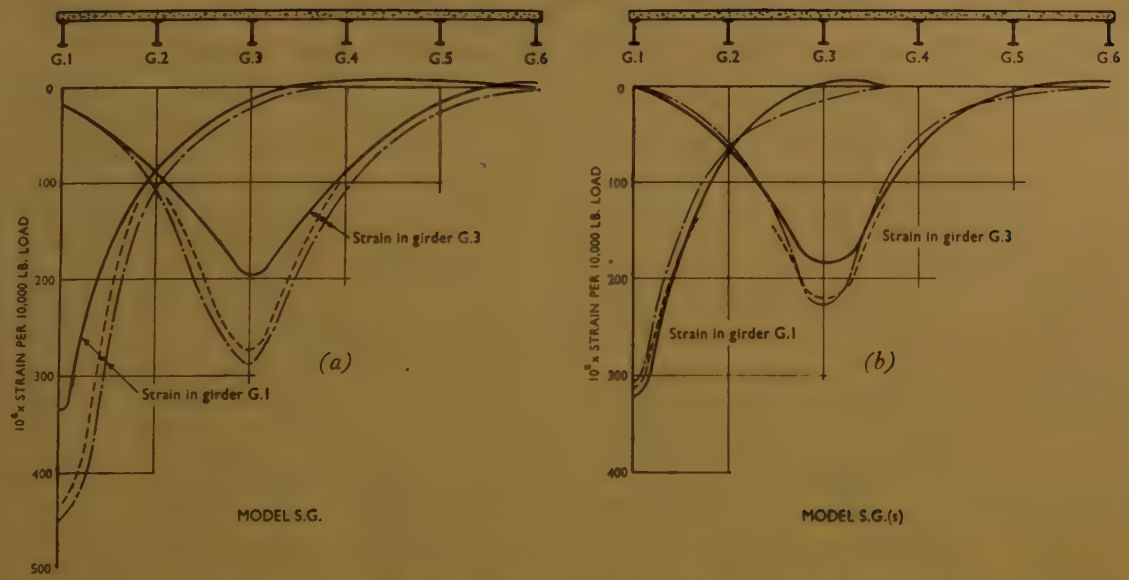
Figs 5



INFLUENCE LINES FOR DEFLEXION AT MID-SPAN OF SLAB-AND-GIRDER SYSTEMS FOR LOAD AT MID-SPAN

(Full lines denote deflexions measured in uncracked models ;
broken lines „ „ „ „ cracked models.)

Figs 6



INFLUENCE LINES FOR STRAIN AT MID-SPAN, IN BOTTOM FLANGE OF GIRDERS, IN SLAB-AND-GIRDER SYSTEMS, FOR LOAD AT MID-SPAN

(Full line denotes strain measured in uncracked model;
broken line „ „ „ „ cracked model;
chain-dotted line denotes strain calculated from the theory of a semi-infinite plate supported on beams).

vehicular traffic, the total stresses may be appreciably less in bridges containing shear connectors.

Comparison of the strains measured in the flanges of all the girders showed that the distribution amongst the girders of the aggregate bending moment resisted by them was not greatly influenced by the introduction of the shear connectors. For any load position, the proportion of bending moment at mid-span, resisted by the girder directly under the load differed by not more than 10 per cent for the two models. In general, the results indicated a slight reduction of the effective distributing capacity of the slab with shear connectors, and confirmed the theoretical conclusions that a change in the relative stiffness of slab and girder exerts a greater influence on the division of the bending moment between the slab and the girders than on the *distribution* of the bending moments between the girders.

It was deduced from the strain measurements that, before cracking, an intermediate girder directly beneath the load in model S.G. resisted about one-third of the total applied bending moment, whereas in model S.G.(s) the loaded girder resisted three-fifths of the total bending moment. After cracking, the corresponding proportions were two-fifths and two-thirds respectively.

With regard to the transverse bending of the concrete slab when the load was at the centre of a panel (see *Figs 7*), shear connectors reduced the maximum strains in the concrete by 30 to 40 per cent before cracking, but only by about 15 per cent after the formation of cracks. As in the case of the general deflexion of the deck system, the provision of shear connectors had an important effect in reducing the deflexion of the slab relative to the girders.

It appears that there is scope for ingenuity on the part of the designer, in striking an advantageous balance between a number of interdependent factors controlling the maximum stresses in the girders. A low ratio of girder stiffness to slab stiffness leads to a reduction of the proportion of the applied bending moment resisted by the girders ; composite action tends to increase the ratio, and hence the moment resisted by the girders, but also increases the section modulus, thus tending to reduce the flange stresses. With regard to the stress distribution within the girder section, the strain measurements indicated, that in model S.G. both flanges were about equally stressed. In model S.G.(s), however, a large part of the concrete slab was forced to act as compression zones of the composite beams, and the neutral axis was in the region of the top flange of the girder. The top flange was therefore understressed and it seems that the conventional I-section is uneconomic in such circumstances ; asymmetrical sections might well be used with advantage, with reinforced-concrete haunches connecting the web with the deck-slab.

Effect of cracking : slab-and-girder system. The systematic cracking of the slabs produced by the application of concentrated loads along the centres

of the panels was intended to simulate the effects of prolonged service and restrained shrinkage.

In model S.G., cracking led to an increase in the maximum strains of 40–45 per cent for loading over the intermediate girders, and 30 per cent for loading over the edge girders (see *Figs 6* and Table 6). The maximum deflexions were increased by similar amounts (*Figs 5*). In model S.G.(s), however, cracking had comparatively little effect on either the maximum strains or the deflexions.

Cracking of the concrete might be expected to produce a reduction in the stiffness of the concrete slab and also a diminution of the effective composite cross-section, leading in theory to changes both in the bending-moment distribution and in the magnitude of the stresses caused by any particular bending moment. The experimental results indicate that, in model S.G., the combined effect of these changes was an increase in the relative stiffness of the girders and the slab (as expressed by the factor K in equation (1)). This increase caused an appreciable transfer of bending moment to the girders to take place after cracking and particularly to the girder immediately under the load. For this model, any changes in the composite cross-section were negligible, since the degree of composite action was small, even before cracking.

In model S.G.(s), however, the stiffness of the slab, compared with that of the composite-girder section, was much lower than in model S.G., in the tests before cracking. The total applied bending moment was therefore largely borne by the girders at this stage, and any further reduction of the stiffness of the slab, due to cracking, would not result in important changes in the division of the bending moment within the system. The effective composite cross-section, also, was only slightly affected by cracking, as shown by the position of the neutral axis.

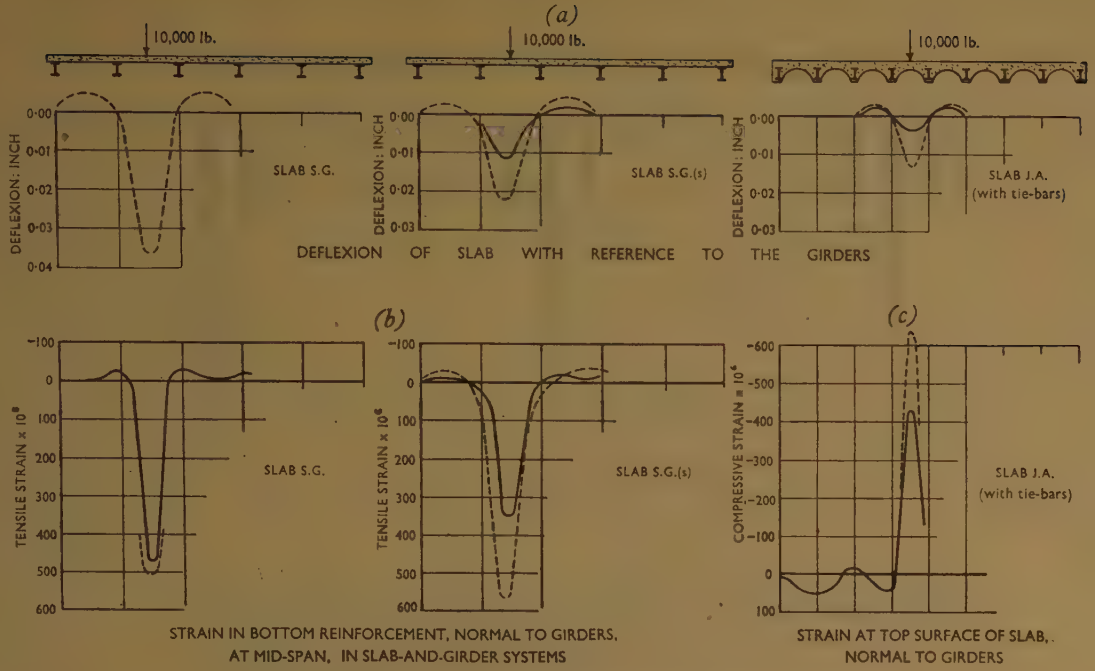
The effect of cracking on the distribution of bending moment amongst the girders was small, particularly for model S.G.(s) (see *Figs 8* and Table 7). The effect on both the strains and deflexions of the beams appeared to diminish with distance from the load.

The transverse stresses in the reinforced-concrete slab were increased considerably as a result of cracking, particularly in the immediate vicinity of a load placed over a panel (see *Figs 7*).

Effect of cracking: jack-arch system.—In the jack-arch model, the successive loading along the centre-lines of the panels resulted in the development of cracks parallel to the beams, near the crown of the arches, instead of the radiating pattern developed in the slab-and-girder systems. Influence lines for the strains and deflexions of an edge girder and an intermediate girder are given in *Figs 9* for the tests before and after cracking. From this diagram, it is seen that the maximum strains and deflexions were increased considerably as a result of cracking.

From the strain measurements on the flanges of the girders, it was possible to estimate with reasonable accuracy the distribution of the

Figs 7



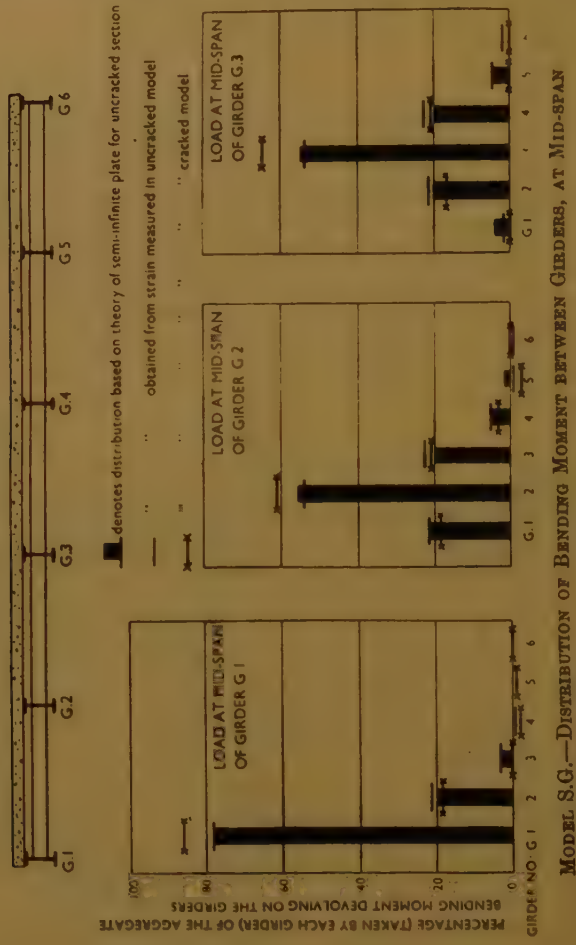
DEFORMATION AND STRAIN AT MID-SPAN FOR A LOAD PLACED AT MID-PANEL

(Full lines denote strain and deflexion in uncracked models
broken lines „ „ „ „ „ cracked models.)

aggregate bending moment resisted by them (see *Figs 10*). Cracking did not affect the distribution considerably.

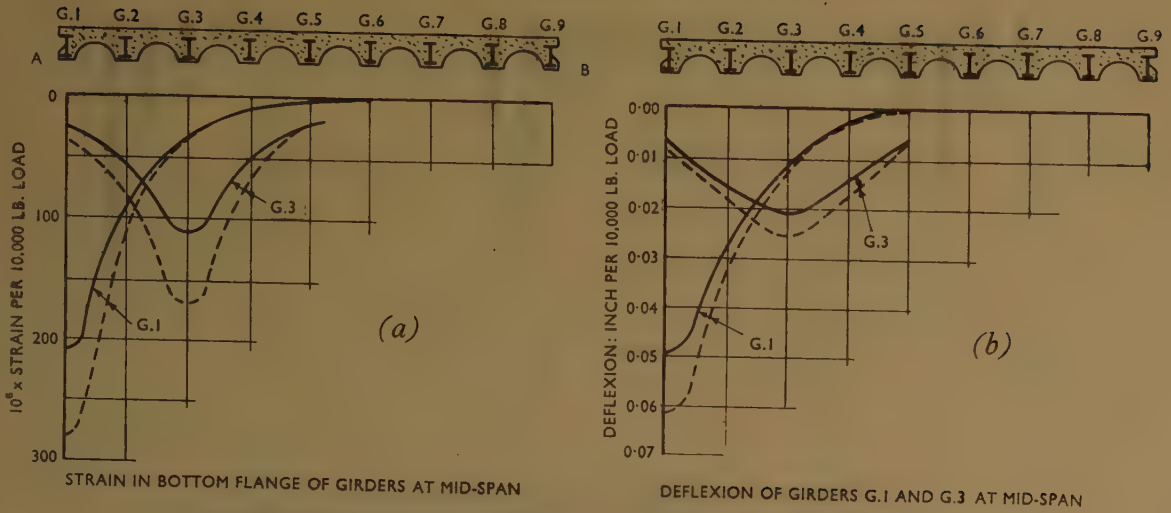
For the estimation of the magnitude, as distinct from the distribution, of the aggregate bending moment resisted by the girders, the shape and

Figs 8


G1
G2
G3
G4
G5
G6
denotes distribution based on theory of semi-infinite plate for uncracked section
" " obtained from strain measured in uncracked model
" " " cracked model
LOAD AT MID-SPAN OF GIRDER G 1
LOAD AT MID-SPAN OF GIRDER G 2
LOAD AT MID-SPAN OF GIRDER G 3
G 1 2 3 4 5 6
PERCENTAGE (TAKEN BY EACH GIRDER) OF THE AGGREGATE BENDING MOMENT DEVOLVING ON THE GIRDERS
GIRDER NO G 1
0
20
40
60
80
100

MODEL S.G.—DISTRIBUTION OF BENDING MOMENT BETWEEN GIRDERS, AT MID-SPAN

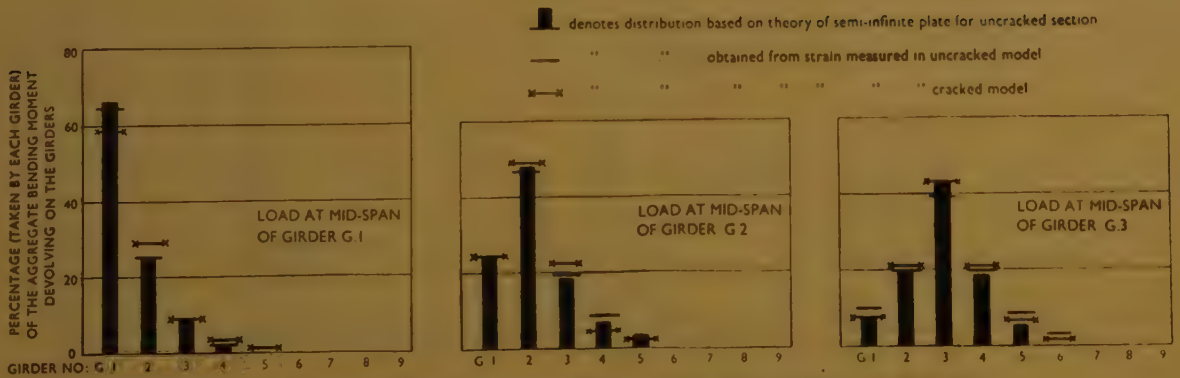
properties of the effective cross-section of the composite girders are required. In the slab-and-girder systems, the effective section could be readily determined, since the slab was of uniform thickness and—in the composite construction—was placed mainly within the compressive zone of the section; the width of slab acting effectively as the flange of the equivalent T-beam was simply related to the measured depth of the neutral axis. For the jack-arch system, however, the thickness of the concrete



INFLUENCE LINES FOR STRAIN AND DEFLEXIONS IN MODEL J.A., FOR LOAD AT MID-SPAN

(Full lines denote strain and deflexion measured in uncracked models ;
broken lines „ „ „ „ „ „ cracked models.)

Figs 10

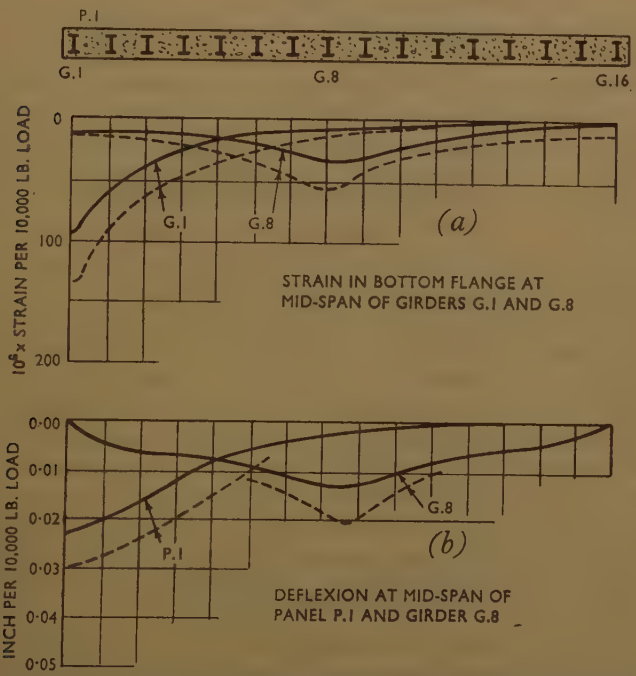

denotes distribution based on theory of semi-infinite plate for uncracked section
" " obtained from strain measured in uncracked model
" " " " " " " cracked model
PERCENTAGE (TAKEN BY EACH GIRDER) OF THE AGGREGATE BENDING MOMENT DEVOLVING ON THE GIRDERS
80
60
40
20
0
LOAD AT MID-SPAN OF GIRDER G.1
LOAD AT MID-SPAN OF GIRDER G.2
LOAD AT MID-SPAN OF GIRDER G.3
GIRDER NO: G 1 2 3 4 5 6 7 8 9

MODEL J.A.—DISTRIBUTION OF BENDING MOMENT BETWEEN GIRDERS, AT MID-SPAN

is variable and part of it is in tension; the strain measurements are therefore not sufficient to determine the effective section unequivocally. It follows that the division of the applied bending moment between the slab and the girders, also, cannot be determined directly as was done for the slab-and-girder systems.

The extreme range of the possible section properties can be calculated, introducing the known position of the neutral axis, which was about 4 inches below the top of the slab, both before and after cracking. For

Figs 11



INFLUENCE LINES FOR STRAIN AND DEFLEXION IN MODEL F.J. FOR LOAD AT MID-SPAN
(Full lines denote strain and deflexions measured in uncracked model; broken lines „ „ „ „ „ „ cracked model.)

the whole range, the possible variation in the theoretical proportion of the applied bending moment that would be supported by the composite girders was from 87 to 98 per cent; and reference to the form of the distribution of bending moment between the girders (*Figs 10*) indicated an equivalent stiffness ratio which corresponded with 90 per cent of the applied bending moment being carried by the girders. If this latter value is adopted as a reasonable basis, both before and after cracking, the measured strains in the girder flanges lead directly to an estimate of the actual effective section modulus.

The transverse strain at the crown of the concrete jack-arch in a loaded panel was more than doubled as a result of cracking. There were similar increases in the deflexion relative to the adjacent girders and in the force in the tie-bar in the loaded panel.

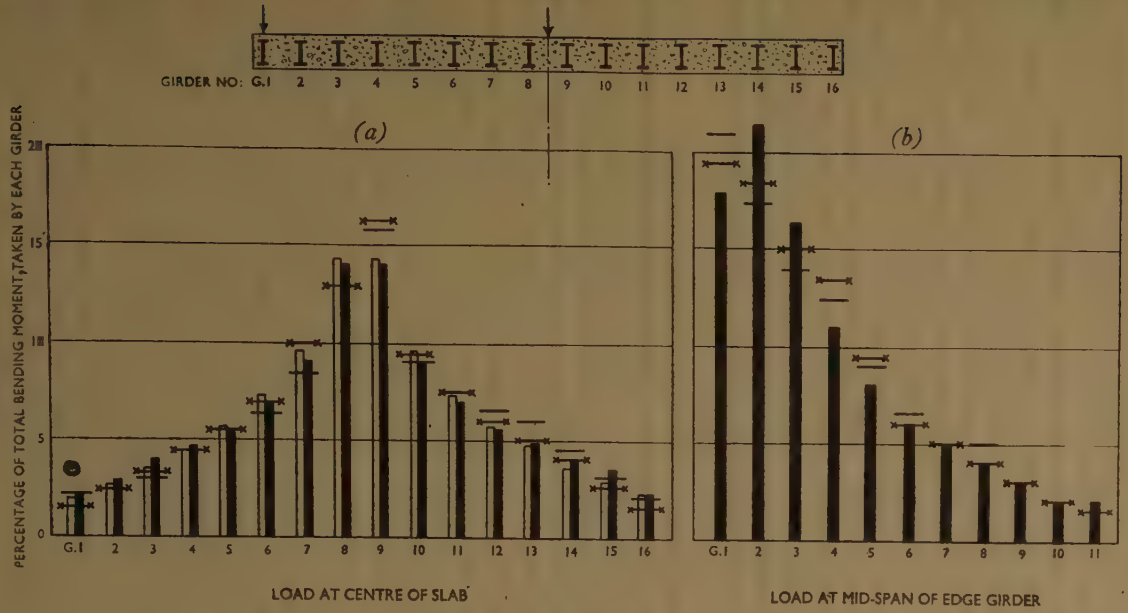
Effect of cracking : filler-joist slab.—Because of the greater proportion of concrete in the filler-joist slab than in the other models, the effect of cracking was correspondingly more important. The maximum strain in the joists increased due to cracking, by about 45 and 60 per cent for loading over an edge joist and near the centre of the slab, respectively. The maximum deflexions were also increased in the same ratios. Whilst the actual maximum deflexions and strains increased rapidly with decreasing distance of the load from an unsupported edge, the effect of cracking on them became smaller (see *Figs 11*). This was particularly noticeable at the edge beam, where the concrete made the smallest effective contribution to the composite section. The influence lines in *Figs 11* show that the effect of cracking was apparent at greater distances from the loading position than in the previous, less rigid, systems of composite construction.

The behaviour of the filler-joist slab closely resembled that of an isotropic plate. The bending-moment distributions derived from the measured strains in the joists are compared in *Figs 12* with those computed from the elastic theory of the isotropic plate. The bending moments per unit width were calculated from this theory, adopting the spacing of the beams as the unit. With this method, the reduced width of concrete associated with the edge joist leads to a low value of the bending moment estimated as being resisted by this joist, so that, particularly for a load over the edge joist, the theoretical distribution is a little unsatisfactory. However, it is easy to devise suitable solutions to this difficulty.

In *Figs 12*, for the distribution of bending moment due to loading at the centre of the system, the results obtained from the mathematical analysis of the anisotropic plate are also given. It will be seen that the distribution so obtained is little different from that calculated for an isotropic plate.

The theoretical position of the neutral axis, calculated from the assumption that the entire concrete section is effective, is 4·6 inches below the top surface of the concrete. In the tests before cracking, the neutral axis was found from the strain data to be at about this depth, except that immediately under the load it was about ¼ inch higher. On the basis of no tension being resisted by the concrete, the theoretical depth of the neutral axis would be about 2·7 inches ; but, for the cracked slab, the measured depth was about 4·0 inches for the joist immediately beneath the load and 4·3 inches for the neighbouring joists. It is clear, therefore, that the concrete section was effective in tension over a considerable part of its depth, even after the systematic cracking of the slab. This conclusion was confirmed by considerations of the section modulus of the slab, computed from relationship (6) on p. 153, putting p_m equal to unity. The section moduli so found indicated that the depth of concrete in effective

Figs 12


GIRDER NO: G.1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
(a)
(b)
PERCENTAGE OF TOTAL BENDING MOMENT, TAKEN BY EACH GIRDER
0
5
10
15
20
G.1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
G.1 2 3 4 5 6 7 8 9 10 11
LOAD AT CENTRE OF SLAB
LOAD AT MID-SPAN OF EDGE GIRDER
denotes distribution based on theory of anisotropic plate
denotes distribution based on theory of isotropic plate
denotes distribution obtained from strains measured in uncracked model
denotes distribution obtained from strains measured in cracked model

MODEL F.J.—DISTRIBUTION OF BENDING MOMENT BETWEEN GIRDERS, AT MID-SPAN

interaction with the middle joists of the system was 8¾ inches before cracking and 7½ inches after cracking.

Effect of position of load relative to the supports.—It was deduced from the strain measurements that changes in the load position relative to the supports had no appreciable effect on the division of the total bending moment between the girders and the slab. The distribution of the bending moment amongst the girders was affected, however, in two ways. First, the proportion of the bending moment, at the cross-section containing the loading position, that was resisted by the girder directly under the load, increased as the load approached a support; and secondly, the proportion of the bending moment at any particular cross-section, that was resisted by the girder under the load, increased as the loading position approached the particular cross-section. These effects were more pronounced for the stiffer slabs. For example, for the slab-and-girder system, the proportion of mid-span bending moment taken by the loaded girder, with the load at quarter-span, was about ⅘ of the proportion when the load was at mid-span. For the jack-arch slab, the corresponding ratio was ⅗ and in the filler-joist slab it was ½.

An interesting feature of the lateral distribution of the effects of concentrated loading is that the form of the variation of bending moment in a particular girder along its length changes with the distance from the load. In a slab-and-girder system, a concentrated load placed over a girder produces a distribution of bending moment in the girder similar to that for an isolated girder with a point load at mid-span; the distribution along neighbouring girders is, however, more nearly similar to that for an isolated girder with uniform loading along the whole length of the girder.

Effect of cross-members, normal to girders.—In some of the working-load tests, the cross-members (transverse diaphragms or tie-bars) were removed, and the effect on the strains and deflexions was determined.

In the slab-and-girder system, removal of the cross-beams caused an increase of from 5 to 15 per cent in the girder stresses and deflexions.

With the load placed over a girder, the stresses in the flanges were uniform throughout the flange-width. With the load placed between the girders, however, there was a tendency for torsional stresses to be developed in the girders. This was more important in systems where there was rigid connexion between the slab and the girders, such as model S.G.(s).

When the cross-members were in place, the mid-span diaphragm introduced restraint to torsional deformation of the girders. The tensile strain in the lower flange of the girder was greatest at the edge adjacent to the loaded panel, at mid-span, but was least at this edge, at quarter-span. When the diaphragms were removed, the tensile stress at the edge adjacent to the loaded panel became smaller than at the opposite edge, throughout the length of the girder.

For the jack-arch slab, the increase in maximum girder stresses, caused by removal of the tie-bars, did not usually exceed 5 per cent in the inter-

mediate girders; in the edge girders, however, the increase was about 20 per cent before cracking and 10 per cent after cracking. The proportionate increases in maximum deflexion were similar to those for stress.

When the load was applied to the crown of a jack-arch, the tie-bars were more important. The total force in the tie-bars crossing the loaded jack-arch was about 3 per cent and 7 per cent of the applied load for mid-span loading on the uncracked slab and cracked slab, respectively. About three-quarters of this total force was borne by the tie-bar immediately below the load. The removal of the tie-bars caused an increase of 50–60 per cent in the strain in the concrete directly under the load with a corresponding increase in deflexion of the loaded arch.

General Discussion of "Working-Load" Tests

The design of a composite bridge-deck system can apparently be based on the consideration of an equivalent system in which a slab rests on girders without composite action. In this equivalent system, the slab is assumed to be the same as in the actual system, but each girder is replaced by a composite girder comprising the actual girder and some part of the concrete slab. With this equivalent system, there appear to be three main factors to be taken into account:—

(1) The allocation of the longitudinal bending moment between the reinforced-concrete slab and the girders.
(2) The distribution amongst the girders, of the bending moment supported by them.
(3) The effective section modulus of the girders.

The first factor is of greatest importance when there is little composite action between the girders and the concrete slab in the actual bridge; where such composite action is considerable this factor could often be neglected altogether.

Guidance on each of the factors can be obtained from the theoretical and experimental investigations briefly described in this Paper. For the slab-and-girder system, the theory made no allowance for the third factor and is therefore not strictly applicable to composite girder sections. Nevertheless, the experimental results indicated that the theory may be applied without excessive error to structures where this factor is of importance, by introducing into the computations an effective girder stiffness corresponding to a composite section comprising the girder and part of the concrete slab or encasement.

If Z denote the effective section modulus of one composite girder (in regard to the maximum strain e in the lower flange of the girder), then the proportion of the total longitudinal bending moment resisted by the girders may be determined experimentally as:

$$p_m = \frac{E_s \Sigma e.Z}{M_{total}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where M_{total} denotes the total applied bending moment ($= Pl/4$ for a load at mid-span).

The bending moment resisted by the girders is distributed between them so that the proportion carried by any individual girder, of effective section modulus Z_i and maximum strain e_i, is :

$$p_i = \frac{e_i Z_i}{\Sigma e Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The bending moment M_i resisted by this composite girder is thus :

$$M_i = E_s e_i Z_i = p_i p_m M_{total} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The effective section modulus may be obtained from the experimentally-determined position of the neutral axis. This presents little difficulty in the case of slab-and-girder systems with no beam encasement. In complicated shapes of section such as that in the jack-arch system, however, some error may be involved because the position of the neutral axis may be the same for a range of possible composite sections. However, for the experiments discussed here, such errors were within reasonable limits.

The theoretical proportion of the longitudinal bending moment supported by the girder is shown in *Fig. 13*, for a range of values for the stiffness ratio K (equation (1)) and for different values of the girder spacing coefficient b (equation (2)). The allocation of bending moment between slab and girders was theoretically dependent mainly on the stiffness ratio, the effect of varying the girder spacing or the position of the load relative to the supports being relatively unimportant. On the same diagram, the results obtained from the measured strains in the girders, using equation (6), are also shown for the slab-and-girder models.

It will be noted from *Fig. 13* that the aggregate bending moment taken by the girders, as indicated by the experimental results, is generally lower than that obtained by the theory, and this divergence is particularly marked where the stiffness ratio is relatively small. The experiments confirmed that the proportion p_m (equation (6)) is not materially affected by the position of the load relative to the supports. As the load approaches a free edge, however, the proportion increases ; and the experimental results were in closer agreement with the theory for this case. It will be seen also from *Fig. 13* that, for loading over an intermediate girder, cracking of the concrete tended to bring the actual bending moment distribution nearer to the theoretical values.

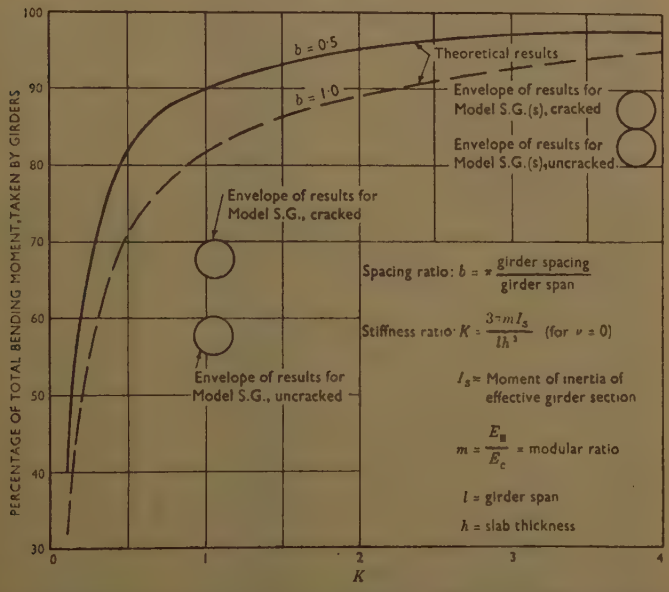
For the jack-arch slab, it appeared from the test results that the theory for the slab-and-girder system could be reasonably applied in the determination of the girder stresses, if the equivalent effective section for the composite girder were known. This effective section can only be established by tests, and on the basis of the data obtained in the present investigation a safe rule for estimating the effective section has been deduced. From

the experiments it was found that it was reasonable to suppose that the ratio p_m was 0·90.

With regard to the lateral distribution of the bending moment amongst the girders, the proportion p_i (equation (7)) of the aggregate bending moment at mid-span, taken by each girder, is illustrated in *Figs 8* and *10*, both for theory and experiment. The comparison shows a fairly good agreement between the theoretical and experimental values.

If it be assumed that the deflexion at mid-span of each girder is proportional to the bending moment resisted by the girder, the distribution of

Fig. 13



DIVISION, BETWEEN THE SLAB AND THE GIRDERS, OF THE BENDING MOMENT CAUSED BY A LOAD PLACED AT AN INTERMEDIATE GIRDER, IN SLAB-AND-GIRDER SYSTEMS

bending moments may be deduced from the deflected shape of the system. In Table 7, the proportion of bending moment resisted by the girder directly under the load is given for the various models on the basis of equation (7) and also as deduced from the deflexions. It will be seen that the assumption of the simple linear relationship between deflexion and bending moment leads to an under-estimate of the bending-moment concentration.

The experiments showed that there was a reduction in bending-moment concentration with increasing distance from the position of the load (see page 152). This effect is of importance when considering the stresses caused by combinations of several loads, but introduces an additional

TABLE 7.—BENDING MOMENT RESISTED BY THE LOADED GIRDER

Measurements from which the bending-moment distribution is deduced	Girder over which the load is placed (at mid-span)	Bending moment resisted by the loaded girder, expressed as a percentage of the aggregate bending moment taken by all girders *							
		Model S.G.		Model S.G.(s)		Model J.A.		Model F.J.	
		Uncracked	Cracked	Uncracked	Cracked	Uncracked	Cracked	Uncracked	Cracked
Flange strains at midspan	Edge girder	78	87	82	83	64	60	21	20
	Intermediate girder	53	65	61	67	42	45	17	17
Flange strains at ¼-span	Edge girder	77	83	77	72	59	55	—	—
	Intermediate girder	46	53	50	52	26	29	—	—
Deflexions at mid-span	Edge girder	77	82	76	79	60	55	—	—
	Intermediate girder	50	59	55	59	30	36	—	—

* *Note:* The bending moment percentages given in this Table were obtained for the models fitted with cross-beams and tie-bars respectively. No appreciable change in bending-moment distribution was observed after the removal of the transverse members, with the load placed over an intermediate girder. With the load placed over an edge girder, removal of the cross-members caused a small increase in the maximum bending moment carried by the edge girder.

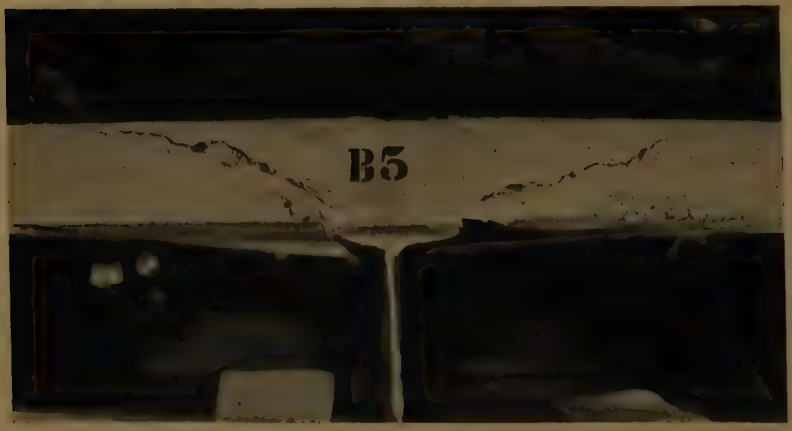
Fig. 14

MODEL S.G. (SLAB-AND-GIRDER SYSTEM, WITHOUT SHEAR CONNECTORS). LOCAL EFFECT, VIEWED FROM UNDERNEATH, OF FAILURE CAUSED BY LOAD PLACED OVER A GIRDER AT MID-SPAN

Fig. 15

Model S.G.(p) (Slab-and-Girder System, with Pre-cast Pre-stressed Slabs). Under-side of Slab after Punching Failures

Fig. 16



Model S.G.(s) (Slab-and-Girder System, with Shear Connectors); Shear Failure at the End of the Loaded Girder

complication if allowance is made for it in design. The use of " effective-width " diagrams, described later in the Paper in the section dealing with design methods, seems to be a simple way of dealing with the matter.

The results for the filler-joist slab suggest that it may be possible to use the elastic theory of the isotropic plate as a basis for estimating the stresses in the joists.

Tests to Failure

Two types of local destruction test were adopted for each model :—

(1) A " punching " test, in which the load was applied to a slab panel between the girders.
(2) A " bending " test, in which the load was placed directly over one of the girders.

The main results are given in Table 8.

" Punching " tests.—Examples of punching failures are illustrated in *Figs 14* and *15*.

The deformation usually increased in proportion to the load although the cracking increased appreciably. At failure, the stresses in the girders were well below the yield-point stress, except in the test on the filler-joist slab.

In the slab-and-girder models, the strain measurements indicated that, when shear connectors were not provided, yield of the slab reinforcement, transverse to the girders, occurred at a load of about 7 tons ; with shear connectors, this reinforcement did not yield until a higher load was reached and the ultimate strength of the slab was increased. Removal of the transverse diaphragms had no effect on the deformations but appeared to reduce the punching strength a little. Failure occurred suddenly, as a result of the severance of a truncated cone of concrete from the surrounding slab ; the angle enclosed between the sides of the cone and its base varied, in a particular test, from 20 degrees to 40 degrees, with an average value for each cone of about 25 degrees. *Fig. 14* shows the type of punching failure in the model without shear connectors (this Figure shows punching resulting from a load over a girder, but the form of slab failure is the same as for a load applied to a panel).

It has been suggested [6] that a useful measure of the resistance to punching failure is the equivalent shear stress computed on the basis that the total shearing force is supported along the surface of a cylinder, the diameter of which is the mean diameter of the cone. For the tests described the equivalent shear stress at failure was about 700 to 800 lb. per square inch. The average diagonal-tension stress was 100 to 130 lb. per square inch at failure.

In one of the subsidiary tests, the slab was pre-stressed and it is interesting to note that in this test the base angle of the cone of rupture was between 17 degrees and 22 degrees ; that is to say, the load was more

TABLE 8.—RESULTS OF TESTS TO FAILURE

Model		Type of test	Load position	Approximate load at yielding of beam or reinforcement : tons	Load at ultimate failure : tons	Bearing pressure under load at ultimate failure : lb. per sq. in.	Angle enclosed by side of punching cone with base
S.G.	With transverse diaphragms	Punching tests	In edge panel	—	9 to 10	2,800 to 3,200	approximately 25°
			In central panel	7	8 to 10	2,500 to 3,200	
		Bending test	Over intermediate beam at mid-span	17	20	6,400	—
S.G.(s)	With transverse diaphragms	Punching tests	In intermediate panels	—	14	4,300	approximately 25°
	No transverse diaphragms			—	12	3,700	
	With transverse diaphragms	Bending tests	Over intermediate beams at mid-span	17	34	10,800	—
	No transverse diaphragms			17	34	10,800	
J.A.	With tie-bars	Punching tests	In intermediate panels	—	16½	5,300	—
	No tie-bars			—	12	3,800	
	With tie-bars	Bending tests	Over intermediate beams at mid-span	18	22½	7,100	—
	No tie-bars			18	18	5,700	
F.J.		Bending tests	In intermediate panels and over intermediate beams, at mid-span	44	47½ to 54½	—	—

widely dispersed than for the ordinary reinforced-concrete slab. A view of the system tested is given in *Fig. 15*. The girders were of T-section and the pre-cast concrete slab units were held in position by pre-stressing, so that the transmission of force between the webs of the girders and the slabs could be effected by friction along the surfaces of contact between them. Pre-stressing parallel to the girders was also introduced so that the structure behaved in the same way as a monolithic slab acting compositely with the girders.

In the punching tests on the main slab-and-girder models, the successive perforation of the slab resulted in a reduction in the resistance to punching of the concrete in neighbouring panels so that the failing load gradually dropped to about 3 to 4 tons. No such effect was observed in the tests on the pre-stressed slab, for which the resistance to punching appeared to be unaffected by previous punching failures.

Tests of the jack-arch system, with and without the tie-bars, showed that the tie-bars reduced the deflexion of the loaded panel, without affecting the girder stresses, and also increased the punching strength of the arch. Failure was again due to the severance of a cone, the base of which was elongated in the direction parallel to the girders.

In the filler-joist slab, the joists were so closely spaced in relation to the depth of the slab that punching shear did not develop as the primary cause of failure, even with the loads applied mid-way between two joists.

"*Bending*" *tests*.—With a load placed directly over a girder, primary failure of all models was caused by yielding of the girders. In the slab-and-girder models, the yield stress in the girders was reached at a load of 17 tons. Where no shear connectors were provided (model S.G.) there was little increase in load before the deformation of the girder allowed a punching failure to develop in the slab above (see *Fig. 14*). In model S.G.(s), however, the composite action between girder and slab, due to the presence of shear connectors, prevented such a failure from occurring separately in the slab, and the load was increased to 32 tons before the loaded composite girder started to separate from the rest of the system. This separation is indicated by the cracking at the end of the girder shown in *Fig. 16*. With a small increase in load, there was considerable deflexion of the girder relative to the rest of the system and the soffit of the slab was forced off, starting at mid-span.

The failure of the jack-arch slab was similar to that of model S.G.(s). Primary failure, due to yielding of the loaded girder, occurred at a load of approximately 18 tons. Subsequent longitudinal cracking at the crowns of the two adjoining arches (see *Fig. 3*) led to virtual separation of the composite girder from the rest of the structure at a load of 22½ tons, and the load then dropped to 13 tons.

In a test with the tie-bars removed, the maximum load reached was reduced to 18 tons but the character of the failure was unaltered.

In the tests of the filler-joist slab, yielding of the joists near the load

position occurred at 44 tons. At a load of about 50 tons, the loaded joist in conjunction with the two adjoining concrete panels was severed from the remainder of the slab; the load-carrying capacity of the separated composite girder was approximately 25 tons. The form of failure and the maximum load were not influenced appreciably by placing the load over a panel instead of over a joist.

The results of measurements of girder strains and deflexions during the "bending" tests are given in *Figs 17*.

Design Methods

In discussing methods of design, it is emphasised that any suggested modifications to present-day methods or the possible use of new methods are merely tentative. Further investigation is necessary before a rational design method can be developed, in collaboration with the Ministry of Transport. The views of practising engineers on the following remarks will, however, be of value in indicating the most useful line of approach in this further investigation.

Present Methods and their Possible Modification

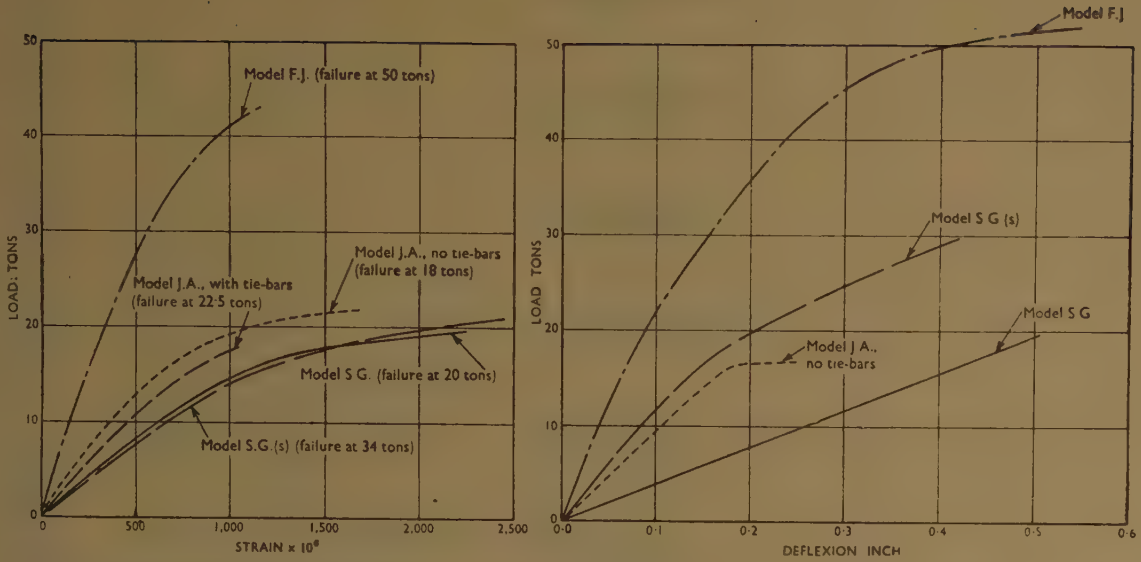
At present, design of steel-concrete bridges is based on the usual flexural theory for individual girders, into which the bridge-deck is dissected and then assumed to be loaded in the most unfavourable manner. The loading adopted may be the standard load-train or the equivalent distributed load developed by the Ministry of Transport; occasionally, the new standard heavy vehicle, with a 90-ton bogey, may also need to be considered.[5]

It is commonly assumed that the concrete takes no part in resisting the longitudinal bending moment; that is to say, composite action is ignored, the role of the concrete being merely to transmit the load to adjacent beams. In filler-joist and jack-arch slabs, however, some allowance for this action is made by increasing the permissible stress in the girders.

A second assumption concerns the distribution of wheel loads between adjacent girders. Where the design is based on the equivalent distributed loading, it is presumed that some allowance has been made for lateral distribution in specifying this loading; it seems improbable, however, that the allowance is wholly satisfactory for all the diverse types of bridge deck. Where the design is based on the standard load-train, the assumptions with regard to the dispersion of load through the thickness of the slab, and the lateral distribution of load amongst the girders, have not been standardized, but are likely to be safe—perhaps unnecessarily so.

The present design method is simple, but it has the disadvantage that it is unsuitable for assessing the actual load-bearing capacity of a bridge, since the design rules are not based directly on the true behaviour of the composite system under load. Accurate assessment of the actual strength of an existing bridge may become important, since the ever-changing

Figs 17

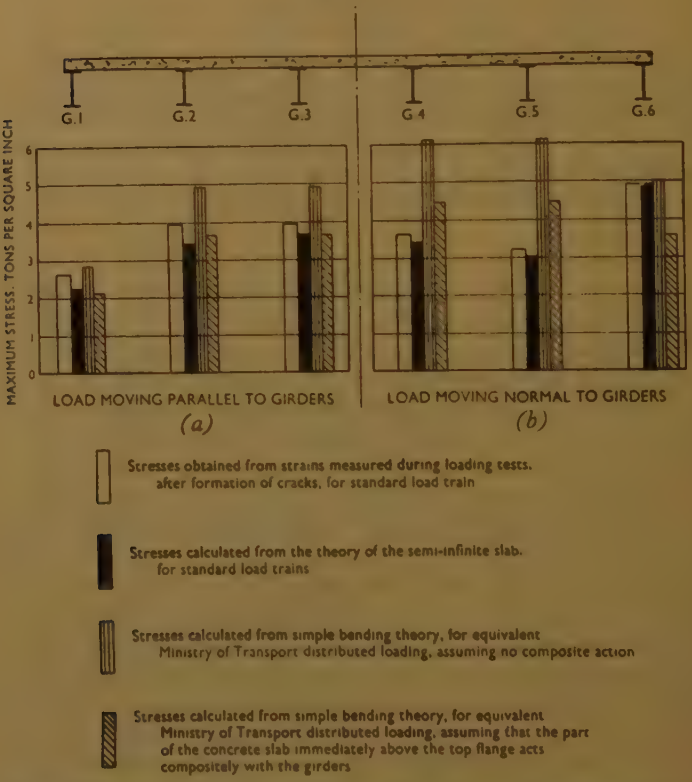


BENDING TESTS TO FAILURE—DEFLEXION AND STRAIN IN BOTTOM FLANGE, AT MID-SPAN OF LOADED GIRDER

pattern of traffic conditions may lead to large modifications in the nature and magnitude of the future loads to be withstood. It is therefore necessary to have a yardstick by which engineers can judge whether a bridge can sustain any new standard of loading with an adequate margin of safety.

It is interesting to compare the experimentally determined stresses resulting from the standard load-train with those computed by the present

Figs 18


G.1
G.2
G.3
G.4
G.5
G.6
MAXIMUM STRESS, TONS PER SQUARE INCH
0
1
2
3
4
5
6
LOAD MOVING PARALLEL TO GIRDERS
(a)
LOAD MOVING NORMAL TO GIRDERS
(b)
Stresses obtained from strains measured during loading tests, after formation of cracks, for standard load train
Stresses calculated from the theory of the semi-infinite slab, for standard load trains
Stresses calculated from simple bending theory, for equivalent Ministry of Transport distributed loading, assuming no composite action
Stresses calculated from simple bending theory, for equivalent Ministry of Transport distributed loading, assuming that the part of the concrete slab immediately above the top flange acts compositely with the girders

MODEL S.G.—MAXIMUM STRESSES IN GIRDERS FOR DESIGN SUPERIMPOSED LOADS

design method based on the Ministry of Transport equivalent distributed loading. Examples of this comparison for the slab-and-girder system (model S.G.) are shown in *Figs 18* (*a*) and (*b*), corresponding to two directions of travel of the load-train relative to the girders. In *Fig. 18* (*a*), it is assumed that the girders are the main girders of the bridge and the load train moves parallel to the girders ; in *Fig. 18* (*b*), the girders are assumed

to be secondary girders of the bridge so that the load-train moves at right-angles to them.

For the intermediate girders (that is to say, all girders other than the edge girders) it is seen from *Figs 18* that substantially larger stresses are obtained by the usual computations than would actually occur in practice. If it were desired to retain the familiar method based on the equivalent-loading curve, either the section modulus of the girder might be assumed to be effectively increased by composite action between the girders and part of the concrete slab, or, alternatively, the equivalent-loading curve might be modified to give better agreement with the experimental results. In the first case, illustrated in the Figures, it is possible to obtain a more accurate estimate of the stresses in the girders ; but it is known that for the particular model chosen (model S.G.) the assumptions as to composite action were not, in fact, realized. It would be irrational, therefore, to adopt a modification to the section modulus, based on composite action, for correcting the design computations.

The alternative procedure of changing the equivalent-loading curve is reasonable, but it is probable that the modification to be made is dependent on the type of bridge deck, since it is essentially a measure of the relative degrees of lateral distribution of load. From the theoretical and experimental investigations that have been made, it may be possible to develop a simple adjustment to the present design method whereby the stresses calculated, both for intermediate and for edge girders, are multiplied by a factor which would be obtainable from tables. Without further study, however, it is not known how simple these tables would be or how many factors besides the basic type of deck would need to be considered.

It should be noted that any such modifications to the present method would not eliminate the disadvantage that it cannot be used for the accurate assessment of the ability of a bridge to resist a specified system of concentrated loads. The bridge engineer may well consider it necessary to use the concentrated load as a basis of design for abnormal loadings ; the introduction of the new heavy vehicle with its 90-ton bogey loads is a pointer in this direction. A consideration of some possible approaches for a new design method may therefore be useful. Should any of these prove of interest to designers, it will be possible to develop the method into a suitable form for practical use, although further research will be necessary before the method can be completely defined.

Design on the Basis of the Elastic Theory.

The investigation of bridge-deck systems at the Building Research Station has shown that the elastic theory leads to a reasonable estimate of the maximum girder stresses if certain assumptions as to the effective composite girder are made. These assumptions are based on empirical data, so that the fundamental analysis must be accompanied by experimental work.

The use of the elastic theory for design purposes involves the presentation of the computations in either tabular or graphical form. Examples of tabular presentation are given, for the slab-and-girder system, in Tables 1–3 for particular values of the parameters K and b. For dealing with a wide range of values of these parameters, the estimation of the mid-span bending moments for mid-span loading appears to require: (1) a main Table giving the bending moment coefficients for an infinite system (as in the first row of Table 3), for perhaps three values of K and of b; and (2) a series of correction Tables (similar to Table 2) for each combination of K and b used in the main Table. From the computations already made, it appears that interpolation between the limited results tabulated would not introduce serious error.

This set of Tables would need to be repeated for loading at other positions relative to the support and for bending-moment distributions at sections other than that at which the load is applied. The minimum number of such sets would probably be five, as follows :—

(1) Load at mid-span, bending-moment distribution at mid-span.
(2) Load at mid-span, bending-moment distribution at quarter-span.
(3) Load at quarter-span, bending-moment distribution at quarter-span.
(4) Load at quarter-span, bending-moment distribution at mid-span.
(5) Load at quarter-span, bending-moment distribution at three-quarter-span.

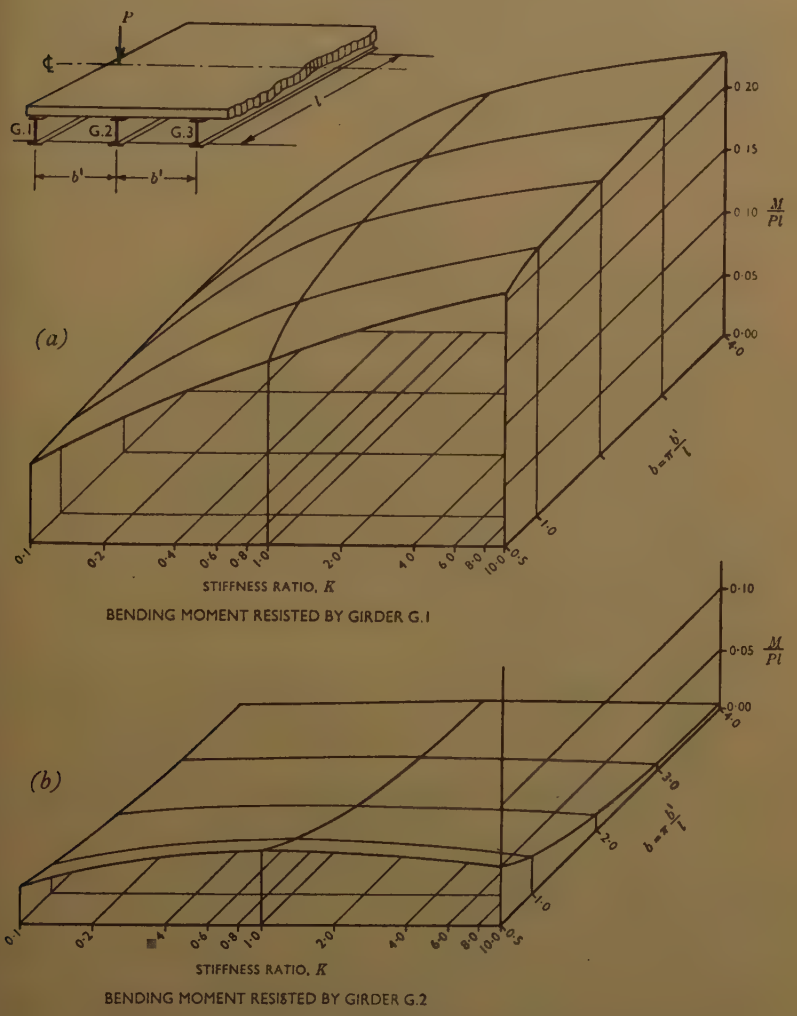
The bending-moment coefficients can be shown graphically in many ways. An example is given in *Figs 19* for a load at mid-span of the edge girder of a semi-infinite system. Each of these diagrams can be pictured as a horizontal base on which is marked the co-ordinate grid for the parameters K and b, with two vertical walls supporting a curved roof, the height of which at a ny point represents the bending moment coefficient M/Pl. It will be observed that the coefficient increases as K or b increases and that, if high values are adopted for both parameters, the coefficient approaches 0·25, corresponding to an isolated girder with a load at mid-span.

To give as much information as the Tables referred to earlier, the number of diagrams of this kind would probably be prohibitive; so that, although *Figs 19* are useful for giving a pictorial representation of the effect of K and b on the bending moments, a simpler method of presentation would be necessary for design. Nomograms might well be the solution if a graphical method is preferred to the use of tables.

The design of filler-joist slabs subjected to concentrated loads may be made on the basis of the elastic theory of the anisotropic plate, by assuming that the bending moment to be resisted by one of the joists is that corresponding to the width of slab equal to the spacing of the joists. It may also be assumed that this width of slab acts in structural combination with

the joist. Apart from the effect of finite width of the system, only one parameter, α, has to be considered; so that design tables may not need

Figs 19



BENDING MOMENTS AT MID-SPAN, OBTAINED FROM THE ELASTIC THEORY, FOR A LOAD PLACED OVER THE EDGE GIRDER G.1, AT MID-SPAN

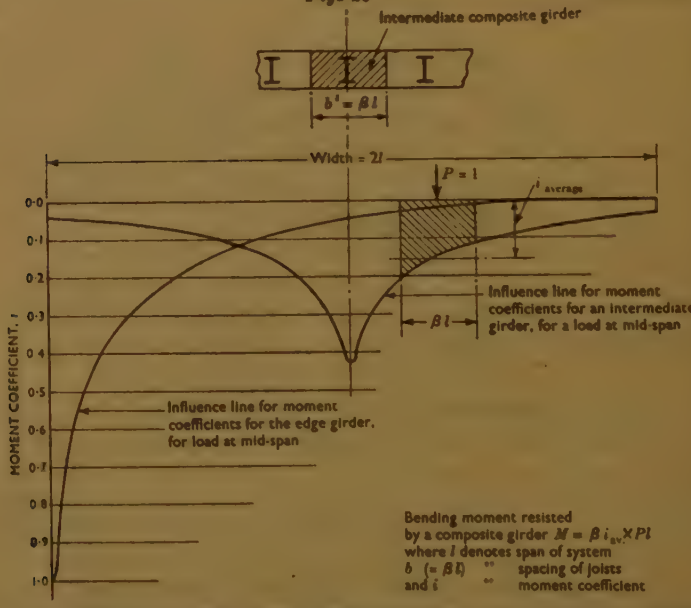
to be so extensive as for the slab-and-girder system. Graphical design methods could be based on simple diagrams of the type shown in *Figs 20*.

It is possible that the theory of the isotropic plate may be sufficiently

accurate as a basis for the design of filler-joist floors, but further analysis is necessary to determine over what range of designs this theory can be applied.

Jack-arch slabs are structurally intermediate between the slab-and-girder system and the filler-joist system, and it may be reasonable to treat a jack-arch slab as an " equivalent " slab-and-girder or as an " equivalent " filler-joist slab. The first alternative has so far received attention at the Building Research Station and in *Fig. 21* an " equivalent " section is proposed which, when used with the theory for the slab-and-girder system,

Figs 20



FILLER-JOIST SYSTEMS.—TENTATIVE DESIGN CURVES FOR BENDING MOMENT AT MID-SPAN

together with an " equivalent " depth of slab, has proved satisfactory for the models tested.

Simplified Rules deduced from the Investigation

It was shown, in the discussion of the working-load tests, that the total bending moment to which a slab-and-girder system is subjected may be first distributed between the slab and the supporting girders. The latter then each resist a proportion of the aggregate bending moment taken by them and the bending moment for which any girder must be designed can be established from these two relationships.

The distribution of the longitudinal bending moment between the slab

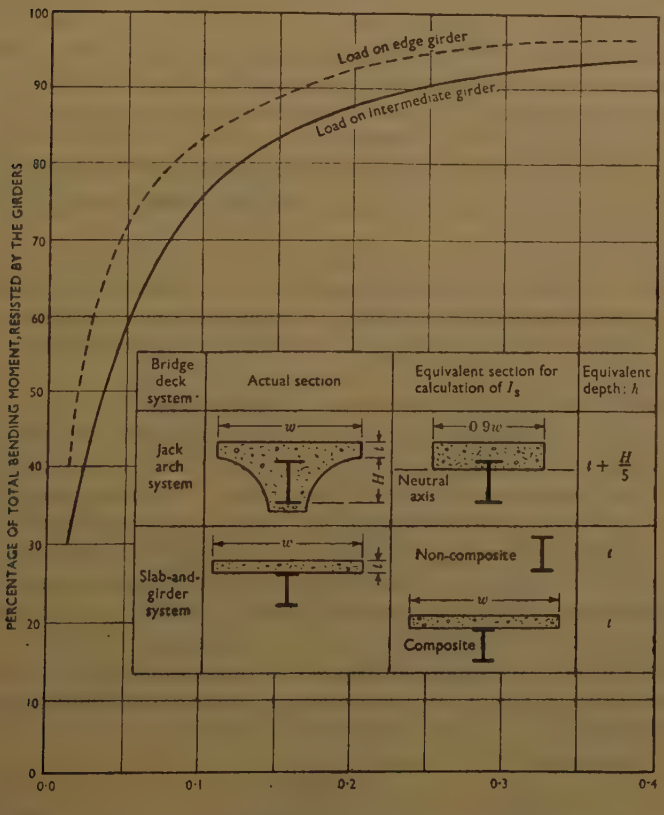
and composite girders is determined mainly by their relative stiffnesses. Instead of the stiffness ratio K used in the mathematical analysis (equation (1)) a simpler ratio $\bar{K}$ may be used for design, where :—

$$\bar{K} = \frac{I_s m}{l h^3} \quad \ldots \ldots \ldots \quad (9)$$

in which $m = E_s/E_c$;

I_s denotes the moment of inertia of the composite-girder section ;
l ,, the span of the girders ; and
h ,, the depth of the concrete slab.

Fig. 21



TENTATIVE DESIGN CURVE FOR DIVISION OF BENDING MOMENTS BETWEEN SLAB AND GIRDERS IN SLAB-AND-GIRDER AND JACK-ARCH SYSTEMS

(In this Figure, the symbol w denotes the spacing of the girders which is denoted elsewhere in the Paper by b')

The proposed relationships are shown in *Fig. 21* for loading on an edge girder, or loading on other girders. The curves were derived from the theoretical relationships, adjusted to take account of the test results. They may be used for slab-and-girder systems, with or without shear connectors, or for jack-arch slabs, using the equivalent sections indicated in the figure. For the jack-arch systems, however, it is probably sufficiently accurate to assume that the equivalent composite sections support 90 per cent of the applied bending moment.

It does not appear necessary to design the concrete slab specifically for the longitudinal bending moment. If it is designed in the usual way to transmit the bending moments normal to the girders, it will almost certainly be adequate to carry the longitudinal bending moment it has to bear.

The distribution of bending moment amongst the girders was found to depend mainly on their spacing, and for practical design it is thought to be sufficient to introduce this spacing as the only parameter other than the position of the load or the section at which the distribution is required. The design can therefore be based on a series of diagrams similar to *Figs 22*. The series should presumably relate to the same five conditions set out on p. 42, but further simplification without excessive loss of economy may be possible. It would be necessary also to introduce modification for systems with only a few girders.

For the filler-joist slab, no simplification of the elastic theory, other than the use of tables or charts, is suggested. As already stated, it may be possible, as a result of further study, to adopt the theory of the isotropic plate as a basis of design rather than that for the anisotropic plate.

The Use of an Empirically Determined "Effective Width"

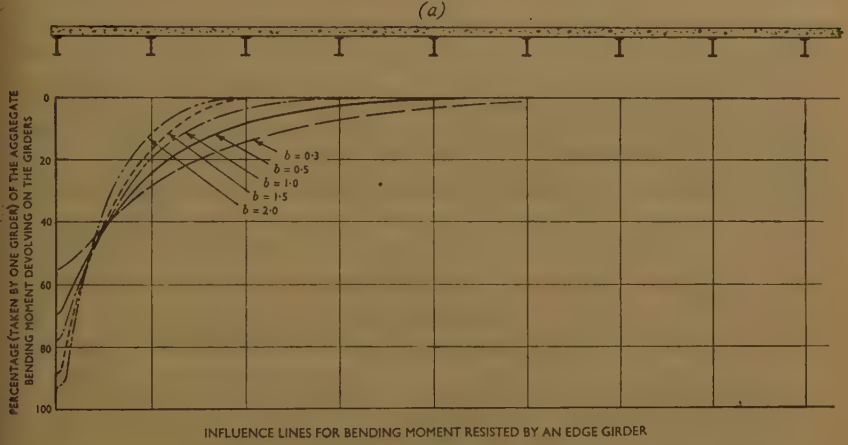
The degree of lateral distribution afforded by a slab can be defined quantitatively by the adoption of an "effective width" of slab over which the load may be assumed to be wholly supported. This method of allowing for the effects of concentrated loading has been adopted, to a limited extent, in a recent Code of Practice, CP 114.1, for Concrete Floors and Roofs. If M_{max} denote the maximum value for the bending moment per unit width at any section parallel to the supports and M_{total} denote the total applied bending moment at that section, due to a concentrated load, the worst conditions of bending would correspond to those which would occur in a slab of width ϵ subjected to the same load distributed uniformly across this width, if

$$\epsilon = \frac{M_{total}}{M_{max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In a composite bridge-deck system of the types discussed in this Paper, an effective width may similarly be used to define the extent of lateral distribution of bending moment amongst the girders, and a Paper on this subject was presented to the Public Works and Municipal Services Congress

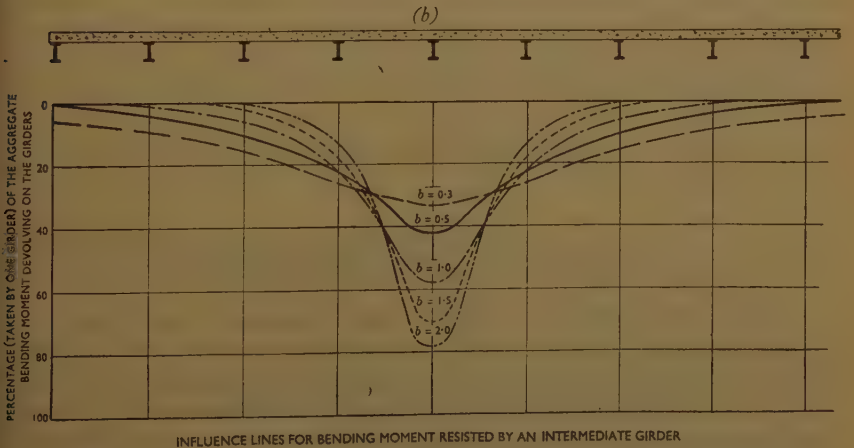
Figs 22

(a)



INFLUENCE LINES FOR BENDING MOMENT RESISTED BY AN EDGE GIRDER

(b)



INFLUENCE LINES FOR BENDING MOMENT RESISTED BY AN INTERMEDIATE GIRDER

TENTATIVE DESIGN CURVES FOR BENDING-MOMENT DISTRIBUTION BETWEEN THE GIRDERS, FOR SLAB-AND-GIRDER AND JACK-ARCH SYSTEMS

in 1950.[7] If the most highly stressed girder resists a proportion p_{max} of the aggregate bending moment borne by all the girders, then the effective width may be defined as :—

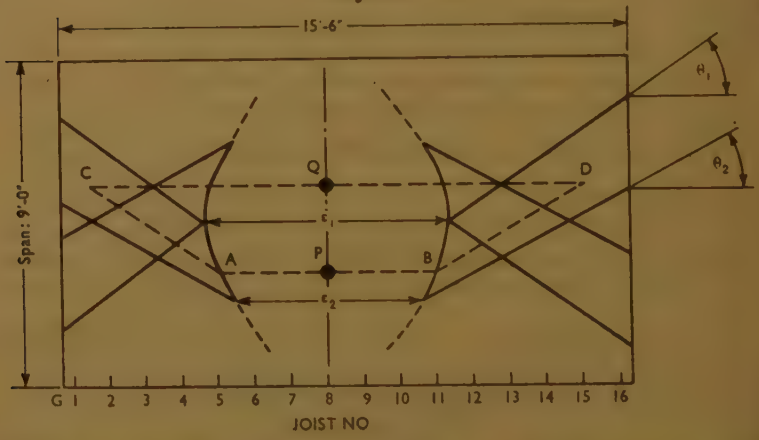
$$\epsilon = \frac{b'}{p_{max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where b' denotes the spacing of the girders.

From equation (7), therefore, if e_{max} denote the strain, in the most highly stressed girder, at the section under consideration, and Σe is the sum of the strains, at this section, in the flanges of all girders, then, assuming that the differences in the section moduli of the girders can be neglected,

$$\epsilon = \frac{\Sigma e}{e_{max}} . b' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Fig. 23



Effective Widths for a Concentrated Load at Various Positions along Joist G.8 of a Filler-Joist Slab (Uncracked Slab)

From this equation, the effective widths can be deduced from the experimental measurements and some results for the filler-joist slab are given in *Fig. 23* for loads over joist number G8. The central curves of the diagram are separated by a distance equal to the effective width for the section containing the load position. For example, ϵ_1 is the effective width for the mid-span section with a load at mid-span, ϵ_2 is the effective width at the quarter-span section for a load at quarter-span, and AB is the effective width for the section through AB for a load at P. It will be observed that the concentration of the effects of the load increases as a support is approached.

For a particular position of loading, the effective width increases with distance, from the loading, of the section considered. *Fig. 23* shows the

results for mid-span and quarter-span loading, the limits of the effective widths being indicated by the lines which start from the ends of the dimension lines for ϵ_1 and ϵ_2 respectively. For mid-span loading, it will be seen that for sections within a distance of about one-eighth of the span from the supports, the effective width is equal to the whole width of the slab; it may be deduced from equation (11) that, within this distance, the joists are all equally stressed.

The rate at which the effects of load concentration become less with distance from the load (in the direction of the span) is defined by the angle θ_1 or θ_2 for loading at mid-span and quarter-span respectively. If θ is not greatly dependent on the position of the load relative to the supports, a simple design basis is possible. To find the effect of a load at P on the bending moment in joist G8 at position Q, for example, the line AB is first sketched and then lines from A and B are drawn at an angle θ with the direction of the supports until the section through Q is reached. The intercept CD is the effective width at this section corresponding to the load at P.

Diagrams similar to that in *Fig. 23* have been deduced from the experiments for other bridge-deck systems and some are given in the paper to which reference has already been made.[7]

Fig. 23 can be used for determining the effect on joist G8 of several loads acting in line across the span, but does not immediately show the effects on other joists. The form of distribution of bending moment amongst the girders is probably related in a fairly simple manner to the effective width; the greater this width is, relative to the total width of the system, the more uniformly is the bending moment distributed. It is possible, therefore, that a simple design procedure for any system of applied loads can be developed, and this possibility is now being studied.

Conclusions

The results of the investigations described in this Paper indicate that :—

(1) There is considerable advantage to be gained in bridge design by allowing for the true structural interaction between girders and concrete developed in modern steel-and-concrete bridge-deck systems.

(2) A safe estimate of the strength of the bridge-deck systems considered can be obtained from the use of the elastic theory, if suitable allowance is made for the stiffening of the girders by composite action. The experimental results suggest possible modifications to the theory to give closer correspondence with the actual behaviour of the systems.

(3) Several approaches to more rational design appear to be possible; and if one of these is chosen a practical design method could

be developed, although further experimental work would be desirable.

Acknowledgements

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The Authors wish to acknowledge the help of several of their colleagues, particularly Mr C. C. Ritchie, M.A., who was responsible for the theoretical investigation of the slab-and-girder system, Mr W. T. Lawton, M.A., who was responsible for the analysis of the anisotropic plate, and Mr R. I. Lewis.

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6. C. Forssell and A. Holmberg, "Concentrated Loads on Concrete Slabs." *Betong*, vol. 31, p. 95 (No. 2, 1946).
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The Paper is accompanied by nine photographs and twenty-four sheets of diagrams from which the half-tone page plates and the Figures in the text have been prepared.

Discussion

The Authors introduced the Paper with the aid of a series of lantern slides.

Mr F. M. Fuller said that he readily accepted all the statements made in the Paper, and he had made an endeavour to see to what extent the present research might affect the design of bridges in so far as his own office was concerned. He felt that the Paper might be regarded as a first step, but a very large step, in an investigation which would give very good results.

As the Authors had mentioned, there was no possible doubt that if engineers wanted to take advantage of this work, some relatively simple formulae must be adopted. He did not wish to criticize the staff that designed bridges—he was one of them himself; but unless there was obtained a formula that was accepted, not only by the designer, but possibly by those in the Ministry who checked the designs, he thought it would not be possible to get very far.

He noticed that in the experiments all the girders had been of uniform cross-section and dealt with point loads. Table 1 showed that the edge girder took a very large part of the load, the second girder appreciably less, and after that the distribution was almost uniform. In practice, however, so far as the edge girder was concerned, it could be subjected to one wheel load, as could all the girders within 10 feet of the kerb; but further than 10 feet from the kerb there could be the effect of two wheel loads of passing vehicles fairly close together. From that he would assume that the edge girder would have to be stiff to cope with those coefficients; the subsequent girders from 2 feet to 10 feet could be less strong, and then it was necessary to double-up in strength to deal with the effect of two wheel loads. But if that were done, there was a departure from the fundamental assumption of uniform girders; had the Authors investigated the effect of stiffening up the girders, particularly the edge girders?

When dealing with bridges in cities, where one of the main problems was carrying pipes and mains and where there were footways on both sides of the carriageway, there was no such thing as an edge girder, because there were girders under the footways, sometimes of even heavier cross-section than roadway girders. Owing to the necessity of providing space for pipes, the lateral connexion between the footway girders and the road girders was not very stiff, but they would have quite an appreciable effect on those rather high factors for the edge girders.

He noticed that, in their illustration of the jack-arch design, the Authors had shown tie-rods between every girder. It had been Mr Fuller's practice for some years to have them only at the outer girders, and he noted from the Authors' results that the omission of the inner tie-rods had no appreciable effect.

The results must necessarily bear on point-loads, but it was far simpler to work with distributed loads; and under the Ministry's system, with distributed loads and a knife-edge, he thought that the standard loading itself had already made some allowance for lateral distribution by allowing the use of a knife-edge. He would be most reluctant to have to design all bridges on point-loads. There would be so many different cases to consider that he felt that the benefit of using these distribution factors would be more apparent than real, because a certain amount would have been taken into account in the knife-edge of the Ministry of Transport loading. He thought that the Authors, although not dealing with the design of a

composite beam on its own, had indicated that when dealing with a knife-edge loading it might be possible to get more economy by investigating fully the effects of the composite nature of the beam rather than by concentrating on the distribution effects from one girder to another. He believed it was suggested in the Paper that no account had been taken of the composite nature of the beam in the past. But it had been done by permitting higher working stresses in the steel. That was presumably such a rough-and-ready method that it might not appeal to the Authors; but it was very simple to apply.

The Authors had also indicated that the top flange of girders in the composite beams was under-stressed, and he wondered whether they had considered carrying out experiments with bulb T's or sections of that description—in other words, reverting to the shape of the old cast-iron girders. Whether bulb T's would be rolled in sufficiently deep sections to enable them to be used in practice, he could not say; but he would be very much interested to see the effect of bulb T's in comparison with joists from the experimental point of view.

In conclusion, he hoped that his suggestion of a return to the same section of girder as the cast-iron girders used 100 years ago would not be taken as an indication of the progress that had been made in bridge design!

Mr R. G. Braithwaite said that, during the past few years, in the course of Continental tours, he had encountered quite a number of bridges of composite construction, and he gave one or two details to show how the Continental practising engineer conceived the idea of composite construction.[1]

It seemed to him that there was an excellent opportunity for one of the recipients of the Culman Travelling Fellowship to take a trip on the Continent in years to come and see how those bridges had behaved under weathering and live-load conditions.

Mr Braithwaite referred to some of the latest bridges on the Continent, built in the remoter parts where materials were difficult to obtain, where technical services seemed to be prolific, and where labour seemed to be fairly cheap and the various trades could be mixed. In those cases, the engineers had made very full use of every technical detail to cut down the use of materials to the minimum. For that purpose they had made their main girders in steel; shear reinforcement being provided by simply welding bracketed cleats to the top chord, as shown in *Figs 24*.

That was significant in view of the Authors' remarks that the I-section was not the ideal section for that type of construction. Such a girder would be no more difficult to fabricate than the conventional type if welding were used. The engineer then proceeded to cast a concrete deck across the girders[2] as shown in *Figs 24*.

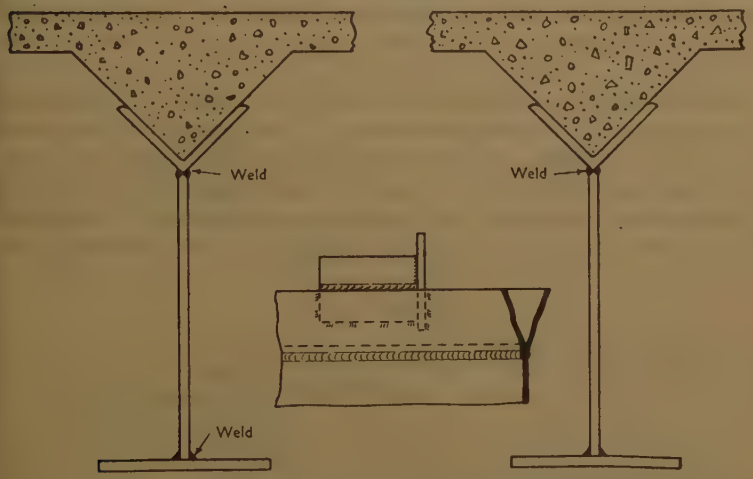
[1] E. Torroja, "Welded Hangars and Bridges in Spain." Trans. Inst. Weld., vol. 14 (Feb. 1951).

[2] D. Steitz, "*Strassenbrucken aus Stahl im Saarland.*" (Steel Road-bridges in the Saar). Bau-Anz. f.d. Saarland, 13/14 (July 1951).

In Mr Braithwaite's opinion, there was, with that sort of construction, the possibility of certain accidents happening. For instance, a shear reinforcement in the angles consisted of a series of plates placed approximately 4 feet 6 inches apart.

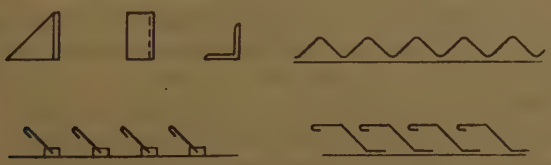
In a previous Paper,[1] he had shown some details of shear connectors,

Figs 24



but on further reflection it occurred to him that a considerable amount of care must be exercised in the use of these. Several of the bridges which he had seen on the Continent had those shear connectors placed in the form of an angle, or just a piece of plate welded to the top flange, or in some cases a T-section shape. (See *Figs 25.*)

Figs 25



They were spaced at distances ranging from 3 feet to 4 feet apart.

In simple girder construction it was not usual practice to use groups of rivets or lumps of weld to attach the flange to the top of the girder at long distances apart; they were spread out according to the shear load. Mr

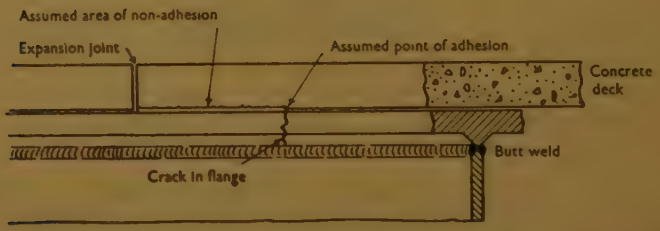
[1] R. G. Braithwaite and D. J. Davies, "Welded Highway Bridges." J. Instn Civ. Engrs, vol. 34, p. 109 (April 1950).

Braithwaite thought that some form of shear reinforcement would have to be devised which did not transmit high local stresses beneath a point-load in the girder ; that was to say, the shear should be distributed uniformly.

He was reminded of a case in the construction of the *autobahnen* bridges in Germany about 15 years ago. There had been certain failures on the then newly developed welded bridges that were being put up. Those bridges were in the form of long girders (probably 30 or 40 metres long—perhaps longer), built as long continuous girders, the top flanges being heavy steel sections of the old " nose profile " type, butt-welded to the webs.

They had been put together on site and built up into a long bridge of continuous girders. When put into service, it had been found that in some girders a series of fatigue cracks occurred through the flanges, and a considerable amount of investigation had been carried out at the time to find out why that had happened. Of course, the new technique of welding had naturally been suspect, and all the limelight had been focused on the

Fig. 26



welding process ; but in the course of a discussion with German engineers, one of the observers had pointed out to Mr Braithwaite a certain peculiar feature in connexion with one of the bridges. The girders carried a concrete road slab. The concrete deck was allowed to rest freely on top of the girder. With expansion joints at 15- to 20-metre intervals and no shear cleats being provided, he had assumed from the design at the time that the girder carried the full load and that the concrete deck was designed purely as a deck (*Fig. 26*). The observer had remarked that all the fatigue cracks throughout the length of the bridge had always occurred at ½-metre or 1-metre distance from the expansion joints in the road deck.

That particular feature had not been investigated at the time, because the metallurgical examination of the steel had shown that the steel was notch-brittle and would have failed in any case when the cold weather came. Nobody had therefore investigated further the question of the peculiarity of the cracking taking place a certain distance away from the expansion joint. The observer had believed that, by some peculiar coincidence, there had been partial adhesion of the slab to the deck beams and, being a continuous girder, there was a combination of reversals of stress, and at a

certain distance from the expansion joint in the concrete slab adhesion had occurred; and there had started, as it were, a notch-effect between the deck and the beam (*Fig. 26*).

That seemed to Mr Braithwaite to be a very far-fetched argument, but it had caused him to think of the possibility of this occurring in composite construction. It might be that the Authors' experiments showed that if there were shear cleats, and the concrete cracked, the bridge would still stand up; but in the course of time, if those joints were not watched, he thought that possibly there would be failure through fatigue. He therefore suggested that when composite-construction designs were contemplated particular attention should be given to the spacing of the shear connexions.

Mr C. S. Chettoe, commenting on the inception of the research which the Authors had described in the Paper, said that about 15 years ago, or rather more, as a result of legislation, the railways had undertaken an investigation into the strengths of bridges carrying roads over their systems, and they had found, using the best methods then available, that nearly 3,000 bridges on classified roads and bus routes were not strong enough to carry 12-ton loads. That was, of course, a matter of great importance to the Ministry of Transport, and it had been felt that it was an appropriate time to ask the Building Research Station to undertake research into the strengths of existing bridges—first, cast-iron girders and jack-arch bridges or girders surrounded by concrete; and secondly, arch bridges.

In respect of the first, the rules had not provided for assistance given by the materials surrounding the girders—that is to say, composite action. (He thought that a certain amount of off-loading was allowed for in the rules.) In respect of arches, there had been no means of taking into account the assistance derived from the spandrel filling and spandrel walls.

The investigation had been started and carried on for some years, and, as a result, what was known as the $\frac{d}{D}$-formula had been arrived at, which allowed substantial relief to live load in assessing the strength of cast-iron girder bridges with jack arches.

It was a rough-and-ready rule, and it was appropriate only for assessing the strength of existing bridges where conditions and the materials surrounding the girders varied. It had been felt desirable, therefore, to start an investigation into the design of new bridges so that more accurate rules could be evolved, if possible, which would enable the structures to be designed more economically. The Building Research Station had been asked to undertake an investigation, and the present Paper was the first result.

He thought that at the present time the only rules which permitted any allowance for the materials surrounding the girders was the allowance of 10 tons per square inch in B.S. 153—that was to say, if the girder was

effectively embedded in concrete, one could allow 10 tons per square inch instead of 9. (Mr Fuller had referred to another rule which must be fairly recent.) But the 10 tons per square inch did not help very much when it was a question of reducing the size of the top flange.

The other question, of course, was the effect of off-loading. In Great Britain the equivalent-loading curve was the standard loading, and the effect of off-loading was perhaps less important with a uniform load over the carriageway, as Mr Fuller had suggested. It only became important, perhaps, when the main structure of the bridge extended on each side of the carriageway under footways or cycle tracks, or where the strength of the edge of the slab or deck had to be considered. But Mr Chettoe thought it was necessary to retain a uniform loading of that kind because it was very easy to deal with and facilitated design ; furthermore, there were so many different types of loading on British highways (and it was necessary also to provide for possible new ones) that it was not practicable to design for point-loads for definite types of vehicles.

There had recently been a very great increase in heavy indivisible loads. There was a new regulation which limited the size and weight of these to 150 tons, and certain main highways in Great Britain were now being designed to that loading. This had to be used as a check for bridges which still would be designed for the old standard loading. To apply that check it was necessary to allow for off-loading because it might be assumed that there would only be one of those loads on the highway at one time.

He hoped that, as the research developed, it might be found possible to evolve a simple method for covering composite action, by using, say, the equivalent moment of inertia of the composite member, allowing for cracking ; and off-loading might be allowed by some such method as the effective width, or one of the other methods referred to in the Paper.

Mr J. E. Jones said that the Paper was likely to be of considerable assistance in making engineers think about what might be done to design more rationally and more economically for the future.

The principles discussed in the Paper were not all entirely new to engineers, but it had taken a long step towards applying some quantitative measure to the guesses and speculations which many engineers had made as to what happened during the course of the loading of a bridge structure.

The line of development which the Paper suggested would, he thought, lead to economy in design. Of course, economy was being emphasized very much to-day, and various short-term expedients were being proposed to further it. But it was also most important to think of economy in the long term. Britain was no longer wealthy and would have to husband its resources more carefully in the future.

The Paper dealt with types of construction which were appropriate to small-span bridges, and the Authors had quoted some figures which Mr Jones had given a few years ago showing the proportion of small-span

bridges in Great Britain. It was quite clear that it was in that field of bridge engineering that the greatest bulk economy was likely to result.

Figs 18 showed that the Ministry of Transport equivalent loading was, on the whole, conservative, except perhaps for edge girders, in the type of structure which the Authors had chosen to illustrate the comparative effects. That equivalent loading did in fact take considerable account of distribution both in the evaluation of the distributed load itself and in the evaluation of the knife-edge loading, and it was probable that it did not go far enough. But certainly at the time it had been issued, in 1931, it had been quite a long step forward from what had been the practice. Mr Jones thought that the Paper showed that that step had been justified at the time, and that in fact it would have been possible to go further.

The Paper did not suggest new design methods, and Dr Thomas, in his introductory remarks, had explained why he had asked for any suggestions to that end; but whatever methods might be developed, they must be simple ones both in form and in application. It was useless to devise complicated methods of design for the sorts of structures discussed in the Paper. Whether the new design methods should take the form of no more than modifications of the existing prescribed design loads, or the adoption of an entirely new type of design load, he had not yet made up his mind, except to say that, having worked with it for some years, he would be very reluctant to abandon the Ministry of Transport equivalent-loading curve.

The Paper gave a good deal of help in dealing with the strength of existing bridges, and it would also help in deciding what could be done with existing bridges, short of complete reconstruction, to strengthen them.

As Mr Chettoe had said, there were lots of heavy loads about, and although Mr Chettoe had referred to a new regulation which was to limit the gross weight of those loads to 150 tons, it should be remembered that there was provision for a waiver of that restriction, and in fact loads in excess of that amount could be permitted to travel. They had to deal today with loads up to 200 tons—not as a regular thing, but more frequently, perhaps, than one cared to contemplate. Therefore, the problem of heavily concentrated wheel and axle loads was likely to occur with increasing frequency.

He wished to have a little dig at Mr Fuller, who had said that he noted with pleasure that the Authors' statements on p. 153 of the Paper supported his idea that tie-bars in jack-arch bridges were not much good. (That was the effect of the statement, although it had not been put in those words.) He (Mr Jones) had always thought that they were, and he felt he was not going too far in saying that the Paper supported him in his view, because of what was said on p. 153 regarding the value of tie-bars when the loads were applied to the crowns of the jack arches.

Mr Jones entirely agreed with Mr Braithwaite that the small steel plate or angle or T-cleat at fairly wide spacing was a very poor shear connector

between slab and girder, and the Paper itself illustrated a very much better arrangement for the transmission of shear.

Mr O. A. Kerensky said that he would respond to the Authors' request for views on possible design methods. His firm had designed several bridges with combined action, and two of them were being built in Siam at the present time. Present design methods had little to recommend them except simplicity. The adoption of combined-action design was most desirable for the following reasons: (1) it conformed to the facts; and (2), it resulted in considerable economy. For example, in the case of slab-and-beam construction using welded beams with unequal flanges, as much as 30 per cent of the steel in beams could be saved.

He proposed to deal with the design of slab and beam decks only, because he had not had much experience of the other types of deck for bridges.

The designer was faced with four problems to which it would be most desirable to get an answer :—

(1) The distribution of load between the various beams supporting the slab.
(2) The bending moments in the slab itself when it acted as a distributor in addition to its normal function.
(3) The strength of combined beam and slab—that was to say, the amount of slab that could be taken with the beam in assessing its strength.
(4) The design of shear connectors to ensure combined action between beam and slab.

Dealing first with the distribution of load to beams, in practice the problem of allocation of moment to each beam was greatly simplified by the type of loading usually adopted in Great Britain. The Ministry of Transport standard loading allowed for uniform distributed load and a knife-edge, and there was no problem as to how much each beam carried, since the whole deck was loaded and there was no distribution of wheel loads. When train loads were used on multiple lanes, the proportion carried by each beam was related to beam spacing. Quite simple formulae could be evolved for that.

As stated by the Authors, relative stiffness of slab and beam did not appear to affect this distribution materially.

There remained only the case of a single load, such as the Ministry of Transport special loading, and for that purpose Tables 1, 2, and 3 and *Figs 22* seemed to provide the necessary information. To simplify the work further, he thought that, in most cases, maximum moments at mid-span only would be quite sufficient to determine the size of the steel beams.

With regard to bending moments and shear in slabs, he thought that some research was required in order to produce some simple rules for maximum moments in slabs in both directions.

When finding the equivalent inertia of a combined beam and slab, guiding rules were needed for the effective width of concrete slab to be taken into account and the modular ratio to be adopted. The effective width in the case of longitudinal girders would, in most cases, be the spacing between the beams, but it was not necessarily so when the beams were spaced far apart. With regard to modular ratio, the Authors had used a value of 6 for calculating the strength of combined beams. The ratio used in ordinary concrete design was 12 to 15, and the Americans recommended 8 as a true ratio for obtaining deflexion, and 10 as a safe ratio for design purposes. He thought that some further guidance on that point was immediately necessary.

The question of shear connectors had already been discussed at length. In order to ensure combined action, shear connectors must always be specified. Guidance was needed for designing them. Although $\frac{Vm}{I}$ gave approximately a correct estimate of horizontal shear between slab and beam, the method of picking up this shear and transferring it to the flange of the beam was in doubt. Two requirements had to be fulfilled: (1) to connect for horizontal shear; and (2) to tie down vertically. Rigid connectors, such as angles, channels, etc., and flexible ones, such as reinforcing bars, hoops, and spirals were used, but the design assumptions and allowable stresses were difficult to arrive at, and further research and practical recommendations were needed.

The American Society of Civil Engineers had published a symposium on Bridge Decks in the 1949 Transactions, which gave semi-empirical rules for the design of combined-action decks which agreed fairly well with the Authors' experimental results, and he had found them quite convenient to use, although some adaptation was required for British loading. No doubt the Building Research Station could evolve improved methods applicable to British Standard loads, and he appealed to the appropriate authorities to put that important work in hand as soon as possible, because the general adoption of that method of design was long overdue.

Dr J. Hajnal-Kónyi said that as a practising engineer—and also as a taxpayer—he was interested in the efficiency of the various types of bridges, and he thought that the data contained in the Paper were most valuable for the purpose of drawing certain conclusions.

The values given in Table 8 for the load at yielding and the load at failure made it possible (after first disregarding the filler-joist type, which had to be considered separately) to compare directly the first three types.

In the bending test, the load at yielding was 17 tons for the slab-and-girder type and 18 tons in the jack-arch type. The jack-arch without ties failed at 18 tons; with tie-bars the load could be increased to 22½ tons. No doubt owing to a re-distribution of stresses, the girder directly under the load was relieved by the co-operation of the adjacent girders; so the load could be increased.

In the case of the slab-and-girder type, the load could only be increased from 17 tons to 20 tons, whereas with the shear connectors the load could be doubled from 17 tons to 34 tons.

All those bridges, he supposed, had been designed on similar assumptions, and so far as failure was concerned the results were totally different. He thought that the conclusion to be drawn from those figures was that neither the jack-arch, even with ties, nor the slab-and-girder bridge, without shear connectors, was an efficient system. In his opinion both those systems should be discouraged and even discarded in future design. Of the three types, it was obviously the slab-and-girder bridge with shear connectors which should be adopted. More research was necessary to find out the most suitable form of shear connectors, but apparently the method used by the Building Research Station had been very successful.

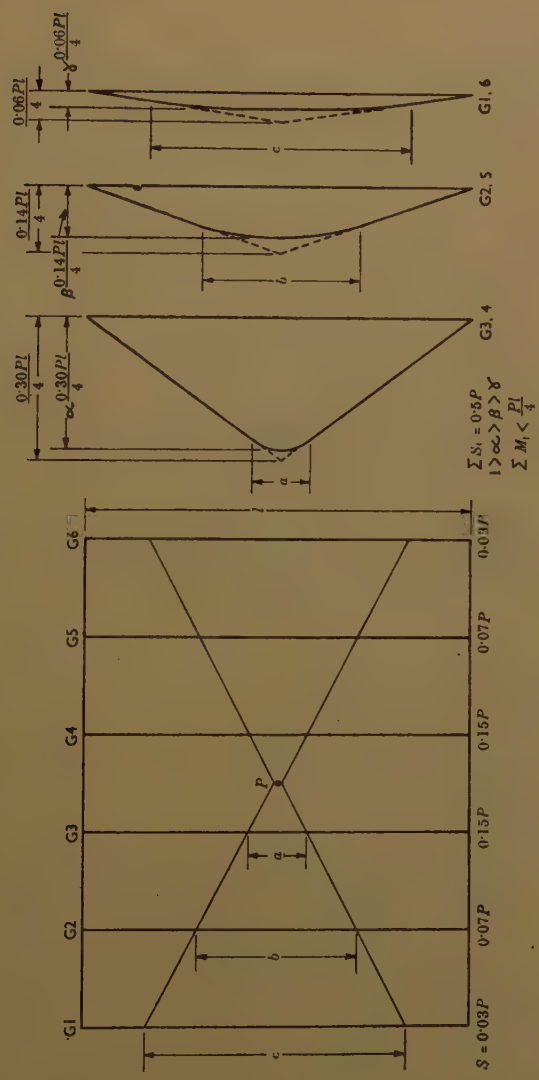
With regard to the filler-joist type, where the failure load was even higher than in the slab-and-girder bridge with shear connectors, he thought that the result would not have been substantially different if, instead of using rolled steel joists, an ordinary solid slab had been used with an appropriate reinforcement. One therefore came back to the solid slab in reinforced concrete, and he did not see what advantage could be obtained with the filler-joist type compared with the reinforced-concrete slab. In fact, he could see many disadvantages with the type as shown in *Figs 1*. Apart from the question of steel consumption the main disadvantage was that the concrete underneath the bottom flange of the joists could hardly be properly consolidated on site; it was likely to be honeycombed, and there was a danger of corrosion. Then, if the steel was fully stressed, cracks would be likely to occur, so that the danger of corrosion would be even greater. On the other hand, if the equivalent steel area was used and subdivided into small round bars, then the difficulty of compacting the concrete did not arise; and it was known that reinforced concrete behaved very satisfactorily.

In view of the fact that steel was now very scarce, he felt that the filler-joist design as shown in the Paper should also be discarded and replaced by an ordinary reinforced-concrete slab. As an alternative, the type of bridge used by British Railways, Eastern Region, and which had been mentioned at previous meetings, seemed to be a very satisfactory solution.

Dr Hajnal-Kónyi then commented on the following statement:

> "In a slab-and-girder system, a concentrated load placed over a girder produces a distribution of bending moment in the girder similar to that for an isolated girder with a point-load at mid-span; the distribution along neighbouring girders is, however, more nearly similar to that for an isolated girder with uniform loading along the whole length of the girder."

That effect was illustrated in *Figs 27*, where an arbitrary distribution of the load on six girders had been assumed so that the girders transmitted



Figs 27

the total load to the abutments, since $\Sigma S_i = \frac{P}{2}$ at each support and $\Sigma P_i = P$. Nevertheless, the bending moment M_i on each girder was less than $\frac{P_i l}{4}$. The relative difference between M_i and $\frac{P_i l}{4}$, characterized by

$1 - \alpha$, $1 = \beta$, etc., increased with increasing distance from P (the position of the load) owing to the distribution of the load by the slab in a longitudinal direction.

He thought that that simple example might help to visualize the co-operation between girders and slab and explained the reason why the total bending moment M_1 on the girders at mid-span was bound to be less than $\frac{Pl}{4}$.

The Authors, in reply, observed that, as was stated in the Paper, they had not put forward firm design rules at the present stage, but merely a number of tentative suggestions with the sole purpose of finding out the reactions of practising engineers to those suggestions, so that they would know what direction they should follow in their researches. They were most anxious that whatever they did should be of service to practising engineers, and they did not want to spend years working out some very complicated mathematical computations only to find in the end that practising engineers did not like such calculations and must have a very simple formula.

The Authors had deduced from the discussion that, although it was desirable to present the results of scientific investigation in a fairly simple form which could readily be applied by the practising engineer, some engineers had already been willing to make the more elaborate computations necessary to obtain the advantages arising from consideration of composite action. It might be that in the near future, using automatic, electronic, and other computing machines, designers would prefer such computations in order to study what really happened in structures, rather than work with the very simple rules of thumb which they had had to accept in the past.

It was interesting that Mr Fuller had found justification in the Paper for his practice of omitting tie-bars except at the outer girders; whereas Mr Jones thought that the Paper substantiated his view that tie-bars were of value throughout the system! The reason for the apparent contradiction was that the experiments had shown that, at working loads, the effect of the tie-bars was small, but, as failure was approached the tie-bars were of value in permitting some redistribution of bending moments between the girders with a consequent increase in the ultimate strength of the system. The usefulness of cross-members generally seemed to be mainly limited to failing conditions in all types of slab. Under working loads the stiffness of the slab was often so much greater than that of the cross-members that the effect of the latter on the distribution of bending moments within the structure was not significant.

When introducing the Paper, the Authors had mentioned that there were two reasons for wanting to know more about the structural behaviour of bridge-deck systems; first, for the design of new bridges, and secondly, for the assessment of the strength of existing bridges; it was the second

feature which was likely to be the more important. It was stated in the Paper that the use of the Ministry of Transport uniformly-distributed-loading method of design should not be readily discarded, although the suggestion was made that it should perhaps be modified. It might be that a series of standard uniformly-distributed-loading curves could be produced. If the present curve was satisfactory for the slab-and-girder type without shear connectors, for example, it was unlikely to be wholly efficient when applied to a type with shear connectors.

The Authors felt, however, that when it was a matter of special loadings —as, for instance, the 150-ton loading which had been mentioned—the checking of a bridge type, which had been designed in the normal way, to assess its ability to carry those heavier loadings, would depend upon the real behaviour of the system. More important still was the question of the future, which Mr Jones had mentioned. The bridges that were being built now should last for a long while. In about 50 years' time, for example, there might be the possibility of 300-ton or 400-ton loads going over them. How would it be possible to assess whether the bridges were satisfactory ? The uniformly-distributed-loading curve would not help ; it would only show that the bridge was satisfactory for the loadings that were *now* considered as normal. The Authors thought it was necessary to formulate some rules of a rational nature for the assessment of the strength of bridges for special loadings, and they considered that the data in the Paper were useful for that purpose.

Dr Hajnal-Kónyi's remarks, on the relative efficiencies of various types of bridge-deck systems, were interesting, but a consideration of such relative efficiencies had not been included in the limited investigation discussed in the Paper, and might involve matters not mentioned by Dr Hajnal-Kónyi. Many factors would have to be balanced before a particular type of bridge deck was chosen, and the relative efficiency of structural behaviour would sometimes have to be subservient to practical considerations related to site conditions, for example. The aim of the Paper had been to give data on the structural behaviour of certain systems, but the *choice* of systems involved much wider consideration.

Mr Kerensky had been very helpful in putting forward the problems of the practising engineer. The information he had given and the questions he had raised merited careful consideration. With regard to the modular ratio to be used in design, the Authors, in computations based on the elastic theory and in their interpretation of the test results obtained within the elastic range, used $m = 6$. That value was obtained from actual measurements of the properties of the materials used. For design purposes, a value of 10 was probably reasonable.

Mr Chettoe and Mr Jones had mentioned the possibility of strengthening bridges. There again, the results obtained in terms of the effect of the slab in distributing laterally the effects of a concentrated wheel load might be of great importance, because they would help in the design of suitable mats

to go over the top of existing bridges to increase the lateral distribution, thus allowing heavier traffic to pass over.

The type of shear connector used in the tests was not suggested as a practical shear connector, but was devised as being likely to develop full composite action between the girders and the slab. Work had been done abroad on various types of shear connectors, but it appeared from the discussion that further work was necessary. Mr Braithwaite and others had rightly pointed out that when shear connectors were used it was desirable to place them so as to conform to the distribution of shear, and that was not difficult.

It appeared desirable to the Authors, before further research work was undertaken on shear connectors, to obtain agreement between designers and steel makers on a practical range of the most suitable steel sections to be rolled for the girders of composite bridge decks, and on the most advantageous shear connectors to be used with those sections. For filler-joist systems symmetrical I-sections were adequate. For jack-arch systems and for slab-and-girder systems, however, asymmetrical ⊥- or bulb-angle-sections appeared to be preferable, with a heavy bottom flange taking the

Fig. 28

CROSS-SECTION OF PRE-STRESSED COMPOSITE SLAB-AND-GIRDER BRIDGE-DECK, TESTED AT THE BUILDING RESEARCH STATION

internal tensile forces and with rigid connexion to the top concrete slab, which would resist the internal compressive forces.

One design developed and tried at the Building Research Station was mentioned in the Paper (see pp. 157–159 and *Figs 15* and *28*). This system consisted of inverted T-sections, with unreinforced-concrete slabs placed between the webs, at the top. The edges of the slabs were butting up against the webs of the adjoining beams and against each other ; they were held by friction created at the butting surfaces by pre-stressing the slab in both directions. It was found that the horizontal shear was well distributed along the web, that pre-stressing increased the resistance of the concrete slab against punching shear, and that full composite action was maintained in the section even immediately prior to failure. The system was also adequately rigid. The elimination of the top flange of the steel girders, of the welded shear connectors, and of the slab reinforcement could result in considerable saving of steel.

References had been made in the discussion to the use of higher stresses to allow for composite action. That, of course, was one way of dealing

with it, but it had little scientific backing, and there could not be a fully economic use over the whole range of combinations of concrete and steel.

The question of the edge girder was one of some difficulty. The Authors had not so far introduced into their tests or into their analysis the effect of a stiffer edge girder. The effects had, however, been studied at Illinois University where much research work had been done on bridge-deck systems of the slab-and-girder type.

Considering the applicability of the results of the investigation to actual full-sized structures, the Authors observed that the tests had been carried out under static loads whereas superimposed loads on bridges were dynamic. That difficulty was dealt with in design by including an allowance for impact in the superimposed design loads, but the magnitude of the allowance seemed questionable in view of recent investigations carried out by the Building Research Station. The task of assessing the loads to be assumed for the design of bridges was complicated by many more uncertainties, however, and an investigation of the wheel loads and the frequency of their occurrence seemed to be of importance, though difficult.

Mr Braithwaite's suggestion for a survey of the behaviour of composite bridges abroad was valuable. A comprehensive survey had already been made in some Scandinavian countries of the cracking and general ageing characteristics of their larger reinforced-concrete bridges. It seemed that there was scope for a similar survey of existing composite steel-concrete bridges in Great Britain. It could be most usefully undertaken perhaps by County bridge engineers.

It appeared from the discussion that no great disagreement existed today amongst engineers on the problems of composite construction. It was known that intelligent application of present experience and knowledge could lead to considerable economies of steel, labour, and money. There were, however, a number of factors involved in the development of composite methods of construction, and it seemed desirable to co-ordinate them. Successful conferences had taken place recently abroad with that end in view and the time might have arrived to bring together those who were interested in composite methods of construction in Great Britain, also to clear the ground for a systematic approach to its design, construction, and manufacturing aspects.

SPECIAL GENERAL MEETING

Thursday, 20 December, 1951

ALLAN STEPHEN QUARTERMAINE, C.B.E., M.C., B.Sc.,
President, in the Chair

The Notice convening the meeting was taken as read. The Chairman stated that, in accordance with the By-laws, any member present who had not already recorded his vote by post could vote in person at the meeting, and that any member who might wish to do so could withdraw his recorded vote and vote in person.

The thirteen Resolutions were taken separately and the results of the postal vote showing large majorities in favour of each Resolution were announced.

Discussion took place on Resolutions Nos 2 and 8. In connexion with the last-mentioned the Chairman stated that he wished to make it very clear that it was not the Council's intention to lower in any way the Standards required of the candidates for what had hitherto been called Section C, but which would in future be known as the Professional Interview. It was intended that the procedure should be precisely on the same lines as in the past. The candidate would still have to produce his drawings, calculations, and quantities, and a report on the work he had done; he would be interviewed and would have to write an essay on a subject chosen by the interviewer. In other words, although the name had been changed, the test would remain the same as it had been in the past.

After the votes of those voting at the meeting had been taken into consideration, the Chairman formally declared each Resolution carried.

An Experimental Investigation of the Aerodynamic Stability of Suspension Bridges with special reference to the Proposed Severn Bridge

by

Christopher Scruton, B.Sc., A.F.R.Ae.S.

(*Communication from the National Physical Laboratory, England*)

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Introduction

Purpose of Investigation

The investigation to be described was carried out on behalf of the Ministry of Transport by the National Physical Laboratory of the Department of Scientific and Industrial Research. The specific purpose of the investigation was to give guidance on the aerodynamic aspects of the design of a proposed road suspension bridge over the River Severn,* and so to assist the Consulting Engineers † in the development of a safe and economical design. The experiments were begun in 1946 and were concluded early in 1951 by tests which provided a final verification of the aerodynamic stability of the preferred design for the bridge. During the course of the investigation, much empirical evidence has been gained which should prove helpful in the design of other suspension bridges. Time was not available, however, for the more fundamental research work which is still needed to elucidate the root causes of the aerodynamic oscillations.

* The crossing proposed is between Beachley and Aust. The bridge will have a centre span of 3,240 feet, and equal sidespans of 1,000 feet each.

† Joint Engineers for the Severn Bridge—Messrs Mott, Hay and Anderson and Messrs Freeman, Fox and Partners.

Only a brief review of the experiments, with a general statement of the more important results and conclusions, can be given here. A fuller summary has been presented to the Ministry of Transport,[1] and detailed results are given in a series of unpublished progress reports [2, 3, 4, 5, 6, 7] by the National Physical Laboratory.

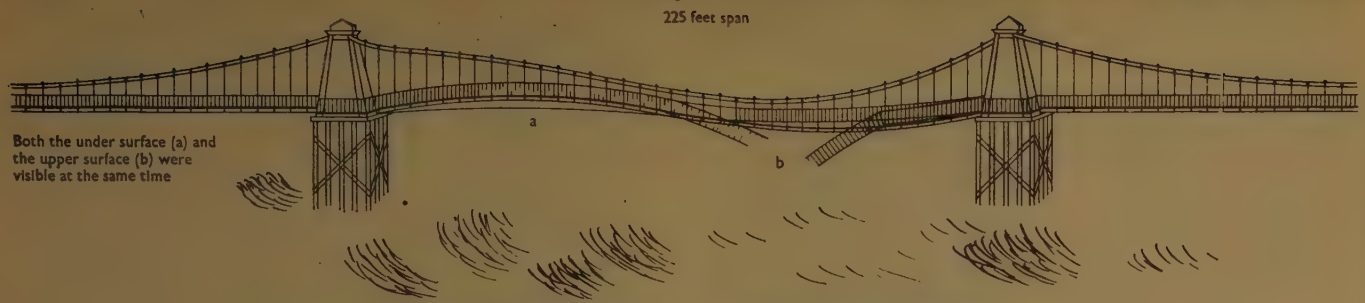
Historical Note

In the past, many bridge failures have been caused by the action of wind. Some of these have resulted from insufficient allowances for static wind pressures but others, on bridges adequately designed to withstand such pressures, have been caused by oscillations of the structure set-up by steady winds. This dynamic action of winds is termed "aerodynamic instability." It will be shown subsequently that the wind speeds capable of exciting oscillations are approximately proportional to the values of the natural frequencies. Hence long-span suspension bridges, with their low natural frequencies, may be expected to be more vulnerable to aerodynamic action than other types of bridge. In fact, the only recorded instances of such oscillations appear to be those involving suspension bridges. An early example is provided by Telford's original suspension bridge over the Menai Straits. One eye-witness stated that the bridge undulated at times in waves 16 feet high,[9] and damage to the bridge was caused by torsional oscillations in 1826, 1836, and 1839. In 1836, one span of the old Chain Pier at Brighton was broken during a storm by torsional oscillations with a node at mid-span [10] (See *Figs 1*). Several modern suspension bridges, mostly in the United States of America, have developed aerodynamic instability but fortunately in most cases it was not destructive.[11] The notable exception was the spectacular failure of the original Tacoma Narrows Bridge [13, 14] by torsional oscillations similar to those which damaged the Brighton Chain Pier (see *Figs 1* and *2*). The Tacoma Bridge, with a main span of 2,800 feet, was only 39 feet wide, and was stiffened along its sides by solid plate girders. During the four months it was in service, vertical bending oscillations occurred in certain winds. These motions were not very large (records show a maximum total throw of about 5 feet) and they did not prevent traffic from using the bridge. On the day of the failure the bridge was destroyed within an hour by the single-noded torsional oscillations which started suddenly in a wind of about 42 miles per hour. The behaviour of this bridge has become widely known through cine-films showing the oscillations and the collapse. These films have a unique scientific value since they have enabled comparisons to be made between the recorded behaviour of an actual bridge with that of models of the same bridge in wind tunnels.

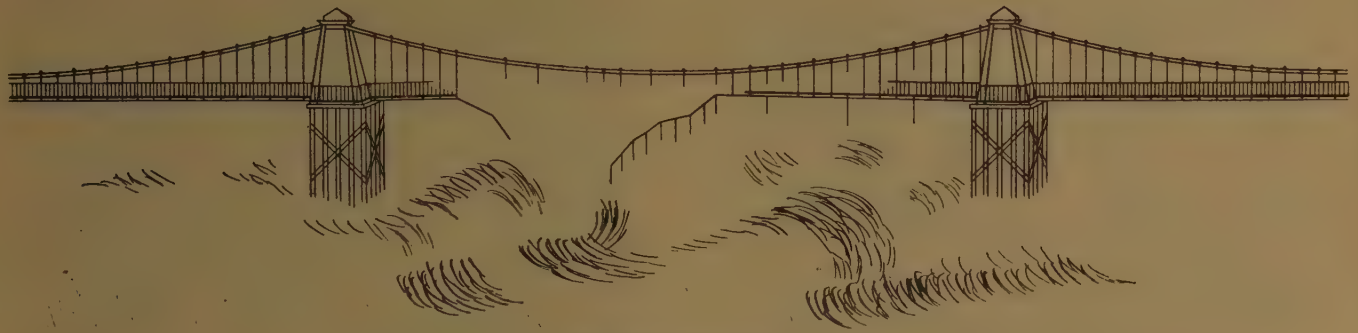
The collapse of the Tacoma Narrows Bridge was followed by extensive researches in the U.S.A. Those at the University of Washington [11, 12] involved building a very large wind tunnel with a working section 100-feet

[1] The references are given on p. 218.

Figs 1



THIRD SPAN UNDULATING BEFORE COLLAPSE IN A STORM ON 29TH NOVEMBER



THIRD SPAN AFTER COLLAPSE

THE TORSIONAL OSCILLATIONS OF THE BRIGHTON CHAIN PIER (1836)
(Copied from a sketch made by an eye-witness)

wide. The first model tested in this wind tunnel represented the original Tacoma bridge and it showed aerodynamic oscillations similar to those which had been observed on the prototype. This good correlation gave confidence in the use of the experimental method, which was then applied systematically to determine a safe design for the reconstruction. The second Tacoma bridge was completed at the end of 1950. Since then high winds have been experienced at the site on several occasions but the bridge has remained satisfactorily steady under all conditions.

Types of Oscillations

The wind-excited oscillations which have occurred on long-span suspension bridges have been basically either vertical bending, or torsional, motions. In vertical oscillations the suspended platform moves up and down and the two cables displace equally and in step. In torsional oscillations the platform twists about a spanwise axis and the cables displace equally but in opposite directions. Both types of motion can occur at various frequencies and in a variety of modes. The instantaneous shape of a spanwise reference line during an oscillation is termed the " wave-form " of the oscillation and is either " symmetric " or " anti-symmetric " with respect to the centre of the bridge, according to whether the displacements of the two half-spans are in the same phase, or in anti-phase. Since the cables can extend only slightly, they must displace horizontally in the direction of the span as well as vertically, in order to maintain approximately constant length. With symmetrical oscillations the cables displace only vertically at mid-span, but the horizontal movement at the towers can be marked. With antisymmetrical oscillations the horizontal displacements can be marked at mid-span, but will in general be small at the towers. With antisymmetrical modes a node must occur at mid-span, and hence the total number of nodes is odd ; with symmetrical modes movement must take place at mid-span, and the number of nodes is even.

The free oscillations of a bridge resulting from an initial general disturbance can be regarded as a mixture of the natural oscillations of the bridge, each of which has its particular frequency, damping rate, and wave-form. The damping rate for each natural oscillation depends on the structural damping and on the damping forces due to the air. The aerodynamic damping is positive when the bridge is in still air but its value is influenced by wind, and for some bridge designs it is possible for the net damping (aerodynamic plus structural) of a natural oscillation to become zero or even negative for certain wind directions and certain ranges of wind speed. In such cases that particular natural oscillation will persist after the bridge has been disturbed in any way. The end points of these speed ranges, which lead to zero net damping and simple harmonic oscillations, are termed " critical speeds." Wind speeds within these ranges yield negative net damping, and therefore the amplitudes of the

Fig. 2

Reproduced from Univ. Wash. Eng. Exp. Stat. Bull. No. 116. (*See reference 11*)

THE TORSIONAL OSCILLATIONS OF THE TACOMA BRIDGE

Fig. 3

THE FULL MODEL

Fig. 5

THE $\frac{1}{100}$-SCALE SECTIONAL MODEL A

Fig. 6

THE $\frac{1}{100}$-SCALE SECTIONAL MODEL B

oscillations will increase initially. Ultimately the motions will either become maintained at a constant finite amplitude (due to departures of the system from linear behaviour) or will continue to grow until structural failure occurs.

A simple physical picture of the way in which wind action may excite oscillations cannot be offered, but some explanatory remarks may be useful. If small, steady oscillations having a constant natural frequency and wave-form were imposed on the system, vortices would be generated periodically and carried down stream, and the aerodynamic restoring-force exerted on the bridge at any instant due to the travelling vortex-system would not in general be in the same phase as the oscillation. It would, in fact, have a negative or a positive damping tendency depending on whether it lagged behind, or acted in advance of, the displacement of the bridge. The strength and the phase of the vortex system would vary with velocity and hence negative aerodynamic damping would be produced only for a range of wind speed. These general considerations do no more than account for the occurrence of instability over definite speed ranges. A more fundamental explanation would require a detailed knowledge of the vortex systems. These appear to be too complicated, especially with a truss-stiffened bridge, for a purely theoretical determination. For this reason, experimental or semi-empirical methods must be used for determining the stability characteristics of a proposed bridge.

Experimental Methods

In the investigation to be described, the oscillatory behaviour of bridges was studied experimentally by tests of models in wind tunnels. The two types of model used were similar to those used at the University of Washington in the extensive investigations which followed the collapse of the Tacoma Narrows Bridge.[11, 12] These were :

(a) Sectional Models

A sectional model is a short rigid model of a sample length of the suspended structure and is mounted across the wind tunnel (with its span horizontal and normal to the wind stream) with freedom to oscillate against spring constraints. In the present investigation the model mountings permitted vertical translatory motions and pitching motions.* These motions, which were the two-dimensional equivalents of the vertical bending and torsional motions of the complete bridge, could take place at the same time for coupled motion tests, or could be isolated for tests with a single freedom.

In general the wind tests of these models involved no more than the observation and measurement of the critical wind speeds and frequencies

* A few tests were also made with lateral motion (that is, translation normal to the span and in the plane of the decks).

bounding the ranges over which oscillations were maintained by the wind. Occasionally the damping rates of oscillations in the wind stream were measured. The tests were made in transverse winds with inclinations varying between $\mp$ 15 degrees. It was not considered practicable to test sectional models in horizontally inclined winds.

(b) Full Models

A full model is a replica of a complete bridge and is so constructed that its behaviour in a wind stream is similar to the full-scale bridge. The full model used in this investigation was installed in a large wind tunnel * specially built by the Ministry of Transport in an aircraft hangar on a disused airfield at Thurleigh, Bedfordshire. The direction of the tunnel wind-stream could not be varied, but the effect of inclined winds, both horizontally and vertically, was simulated by inclining the model. Critical wind speeds, frequencies, and oscillation modes were recorded.

In the early stages of the investigation it was uncertain whether the stability of a complete bridge could be predicted satisfactorily from experiments on a sectional model alone, since with this method of test the influence of several factors cannot be represented directly. Such factors include, for example, the tower stiffnesses and inertias, the longitudinal camber, the oscillation wave-form, and the horizontal inclination of the wind. However, it was also clear that full models were unsuitable for routine comparisons between different forms of suspended platform, owing to the length of time required for construction and the high cost. To provide a practical programme, it was therefore decided to depend on tests of sectional models for an indication of the most promising structural forms. Whilst the tests were in progress, the design and construction of a full model was also put in hand, with a view to tests of the correlation between the two different experimental methods. This full model was necessarily based on a very early design for the bridge, and it became available for wind-tunnel tests in 1948. The results obtained with these two types led to the conclusion that sectional-model tests were sufficient for reliable full-scale prediction, and accordingly the construction of a further full model, based on the design finally preferred for the bridge, was considered to be unnecessary. A final verification of the stability of this design was, however, provided by tests of a large-scale sectional model. An increase of the linear scale from $\frac{1}{100}$ to $\frac{1}{32}$ was considered advisable, because previous tests had shown that the stability was sensitively influenced by details of the structural form which could not be copied with sufficient accuracy on a small-scale model.

Stiffness and Frequency Calculations

Reliable estimates of the natural frequencies and elastic stiffnesses are required in the design of both full-scale and model bridge structures, and

* The wind tunnel is briefly described in Appendix II.

for the interpretation of model-test results. A discussion of the various theoretical methods available is outside the scope of this Communication. As part of the Severn Bridge investigation, a novel treatment has been developed by Dr R. A. Frazer, F.R.S., of the National Physical Laboratory. It is less exacting and time-consuming than a full structural analysis but more rigorous than methods relying merely on simple beam theory. This mathematical work will be published separately.

Static Loads Due to Wind

Another side of the aerodynamic investigation was concerned with the static loads exerted on the bridge structure by wind. This information was requested by the Joint Engineers to assist their design calculations. The necessary wind-tunnel tests were carried out at the National Physical Laboratory on a $\frac{1}{32}$-scale model representing the final design.[7] The model was made in three lengths separated by small gaps. Loads were measured on the central portion only, while the two end-pieces served to ensure the correct airflow. Values of the three component forces and the three component moments were obtained for the complete range of horizontal wind-inclination and for a wide range of positive and negative vertical inclinations.

Natural Winds

In the study of wind action on a suspension bridge, information of a statistical nature on the winds to be expected on the site is very desirable. An effort to obtain such information is being made by the Joint Engineers for the Severn Bridge, with the use of special anemometers which have been developed by the Meteorological Office. The instruments automatically record the wind speeds and the horizontal and vertical inclinations at several positions simultaneously.

For the purpose of wind-tunnel tests on bridge models it is usual to suppose that the wind is steady and distributed evenly along the span, and to place reasonable limitations on the wind speeds and wind directions. In the case of the Severn Bridge investigation it was asssumed that wind of up to 100 miles per hour could be incident from any quarter and with a vertical inclination ranging between $\mp$ 15 degrees. Only in a few special tests was any attempt made to simulate gusty winds.

It is, of course, extremely unlikely that a natural wind would ever be maintained evenly and steadily across the entire span of a long bridge, except perhaps for short periods. However, since the stability of a bridge section is influenced by both wind speed and wind direction, the most unstable condition for the complete bridge may be assumed to be when it is exposed to the most unfavourable combination of wind speed and wind direction at all positions along the span. In this respect then, tests of model bridges in wind tunnels should give results erring on the safe side.

General Conclusions from the Full-Model and the Sectional-Model Experiments

The investigation has shown that reliable predictions of the stability of any proposed design of truss-stiffened suspension bridges cannot, in general, be made without the help of wind-tunnel tests on models. The results of the investigation can be used to obtain guidance on the choice of a design, but if any important new features are to be incorporated, confirmatory tests of the stability are advisable. For this purpose experiments with sectional models are considered to be adequate.

Truss-stiffened bridges are stable in vertical motion and their tendency to instability in torsional oscillations can be corrected by suitable design of the suspended structure. Design features which have a corrective influence are :—

Stiffening-truss chords of high width/depth ratio ;
Separation of traffic lanes by open slots or gratings ;
Truss-type deck stringers in preference to the plate-type ;
Castellated handrailing, or other types of handrailing designed to break up the continuity of the airflow pattern ;
Sidetracks (for example, footpaths, cycle-tracks, etc.) mounted outboard of the stiffening truss.
Lattice-type wind bracing near deck level.

The stability properties of suspension bridges stiffened by plate girders were not studied in any detail. Such bridges are liable to both vertical oscillations and torsional oscillations, and their bluff aerodynamic shapes do not appear to favour the same high degree of stability as is shown by truss-stiffened sections.

Notation

ρ denotes air density.
ν ,, kinematic viscosity of air.
B ,, width of bridge between stiffening trusses.
g ,, acceleration due to gravity.
σ ,, typical material density.
E ,, typical elastic modulus.
θ ,, angular torsional displacement of suspended platform at any instant of oscillation (radians).
z ,, linear vertical displacement of structure at any instant of oscillation.
I_θ ,, torsional moment of inertia per unit of spanwise length.
I_z ,, mass per unit spanwise length.

N_θ and N_z denotes natural frequencies of oscillation in torsional and vertical bending modes respectively.

e_θ and e_z ,, elastic stiffnesses corresponding respectively to I_θ, N_θ and I_z, N_z.

δ ,, natural logarithm of the amplitude of successive cycles of oscillation (logarithmic decrement).

δ_θ and δ_z ,, logarithmic decrements for torsional and vertical oscillations in still air respectively.

$\delta_{\theta A}$ and δ_{zA} ,, logarithmic decrements due to still air damping.

$\delta_{\theta s}$ and δ_{zs} ,, logarithmic decrements due to structural damping.

$\delta_{\theta w}$,, logarithmic decrement of torsional oscillations due to the wind stream.

V and V_c ,, wind speed and critical wind speed respectively.

N and N_c ,, frequencies of oscillation corresponding to V and V_c respectively.

V_r ,, reduced velocity $\left(= \dfrac{V}{NB}\right)$

$$V_s = \frac{V}{N_\theta B}$$

α denotes angular inclination of wind to the bridge platform in a vertical plane—upwinds, positive (degrees).

β ,, angular inclination of the wind to the bridge in a horizontal plane ($\beta = 0$ and 90 degrees for transverse and longitudinal winds).

Model and Full-Scale Similarity

(i) *Full Models*

The motions of similar-shaped suspension bridges in wind may be influenced by the bridge size (B), by the density, damping, and elasticity of the bridge structure (σ, δ_s, E) and by the viscosity, density, and velocity of the air (ν, ρ, V).

By the usual principles of dimensional analysis these parameters can be grouped in the following independent non-dimensional combinations :

$$\left.\begin{array}{l} (a)\ \dfrac{\sigma}{\rho}\ \text{(density)} \\ (b)\ \dfrac{E}{\rho V^2}\ \text{(elasticity)} \\ (c)\ \delta_s\ \text{(structural damping)} \\ (d)\ \dfrac{gB}{V^2}\ \text{(gravitational)} \\ (e)\ \dfrac{VB}{\nu}\ \text{(viscosity)} \end{array}\right\} \quad \ldots\ (1)$$

Complete dynamic similarity between model and full-scale is achieved if the model is geometrically similar to full-scale and the above numbers are equal for both systems.

For tests in atmospheric wind tunnels of practicable dimensions and wind speeds, the full-scale value of (*e*) cannot be achieved. The velocity scale would, moreover, be incompatible with that required by (*d*). However, there is evidence that the aerodynamic forces acting on bluff bodies such as bridge structures depend mainly on pressure action and only indirectly on viscous action, and hence the validity of full-scale prediction by model-testing is not seriously impaired by incorrect scaling of the viscous forces.

The design and construction of a model to accord with the similarity numbers (*a*) to (*d*) presents considerable difficulties. It will be found generally that materials of construction do not exist which possess the requisite combinations of density and elastic moduli, and that even if such materials were available, the exact small-scale reproduction of details might be impracticable. However, it is sufficient if the correct overall inertias and elastic stiffnesses are reproduced in the model. Equivalent similarity numbers to those given in (1) but using inertial and elastic-stiffness terms are :

$$\left.\begin{array}{l} (a)\ \dfrac{I_\theta}{\rho B^4},\ \dfrac{I_z}{\rho B^2},\ \text{etc.} \\ (b)\ \dfrac{e_\theta}{\rho V^2 B^2},\ \dfrac{e_z}{\rho V^2},\ \text{etc.} \\ (c)\ \delta_{\theta\theta},\ \delta_{zz},\ \text{etc.} \\ (d)\ \dfrac{gB}{V^2} \end{array}\right\} \quad \ldots\ldots \quad (2)$$

With a linear scale of $\frac{1}{n}$ the foregoing numbers yield a velocity and a frequency scale of $\frac{1}{\sqrt{n}}$ and $\sqrt{n}$ respectively. The values of V_r and V_s are therefore the same for the full-scale bridge as for the model.

(ii) *Sectional Models*

With strict inertial scaling.—True similarity conditions are obviously not observed in the sectional-model method of test. The use of the method for full-scale prediction assumes that oscillations of a complete bridge arise from the aerodynamic action on the suspended structure alone and that the other components (for example, cables) contribute to the dynamic properties only. It also implies that critical values of V_r are not influenced by oscillation wave-form.

The inertial coefficients of sectional models represent the total contribution of suspended structure and cables. The model-stiffnesses are merely

those which provide the required oscillation frequencies and need not, as the case of full models, be derived from correctly proportioned gravitational and elastic forces. When (*d*) is omitted, and expressions for natural frequencies are substituted in (*b*), the similarity numbers for sectional models become :

$$\left.\begin{array}{ll} (a) & \dfrac{I_\theta}{\rho B^4}, \dfrac{I_z}{\rho B^2} \\ (b) & \dfrac{V}{N_\theta B}, \dfrac{V}{N_z B} \\ (c) & \delta_{\theta s}, \delta_{zs} \end{array}\right\} \quad \ldots\ldots (3)$$

where I_θ, I_z now represent overall values.

Thus the actual velocity and frequency scales for sectional models are interdependent, but one or other may be chosen at convenience.

Inertial scaling not attempted.—In a previous Paper,[15] which describes the preliminary stages of this investigation, tests of sectional models involving isolated motions only were considered. The requirements for similarity were then approached through the solutions of the equations of motion. Similarity conditions equivalent to those given in (3) were, of course, obtained ; but in addition, it was shown that when $N_\theta = N_c$ approximately, strict inertial scaling is not essential and the critical values of V_r depend only on the geometric shape of the structure and on the product of (*a*) and (*c*), that is to say, on $\dfrac{I_\theta \delta_{\theta s}}{\rho B^4}$ and $\dfrac{I_z \delta_{zs}}{\rho B^2}$ for angular and linear motions respectively.

Procedure for Prototype Prediction from Sectional-Model Test Data

Some remarks may be useful on the procedure in the two cases where strict inertial scaling of the model is observed, or is not attempted and $N_c = N_\theta$ approximately.* The critical values of V_r obtained from the model by both these methods are applicable to the prototype, provided the values of $\delta_{\theta s}$ satisfy the stated conditions.† Strict inertial scaling is essential for values of V_s (and N_c) to be applicable : critical speeds are then determined from critical values of V_s or V_r by either of the relations $V_c = N_\theta B V_s$ or $V_c = N_c B V_r$. The value of N_c required by the latter relation is not given by the other method, and it is therefore necessary to use the approximation $V_c = N_\theta B V_r$. Experience obtained with various types of bridge section indicates that this approximation introduces no serious discrepancies within the range of normal winds.

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* For convenience, pitching oscillations only are discussed here. The procedure applies equally well to other motions.

† Some further remarks in connexion with the determination of structural damping are given in Appendix I.

The Full-Model Experiments

Model Construction *

The $\frac{1}{100}$-scale full model is illustrated by *Fig. 3* (facing p. 192). It represented a truss-stiffened bridge of 107-foot width with a 3,000-foot centre span and two equal 1,020-foot sidespans. The cable sag/span ratio was 1 : 10. Section A, of *Fig. 4*, shows in outline the section of the mid-deck type of suspended structure. The two roadways, each 39 feet wide, were separated by an 8·3-foot-wide reservation. The dead loads due to the suspended structure and to the suspension were respectively 5·07 and 1·43 tons per foot span.

The model was intended for research purposes and was designed in such a way that the arrangement of the suspended structure and its dynamical properties could be altered. The required stiffness and inertial properties of the suspended structure were obtained by the use of light rigid components interconnected by steel springs. These rigid components were mostly of aluminium balsa-wood sandwich construction and each spanned the 60 feet between adjacent cross-girders. Since the deformations occurred mainly through the steel springs, this construction enabled the elastic stiffness to be varied by substitution, and also prevented the structural dampings from becoming excessive. The roadways were not continuous in each span, but were divided into segments separated from one another by small gaps at the cross-girders. The supports for these segments consisted of thin vertical steel blades fixed centrally to the two end cross-girders concerned, with their planes at right angles to the span. The resistance offered by those mountings to displacements of the bridge structure was due to the bending or the twisting of the thin blades, and was relatively very small.

Both the elastic and inertial properties of the towers were variable but it was not considered necessary to reproduce the correct external shape of either the towers or the anchorages. The model cable consisted of piano wire of 0·024-inch diameter which provided the equivalent of a full-scale cross-sectional area of about 450 square inches with a $\frac{1}{100}$ reduction of Young's modulus. To obtain the correct mass and external shape, hollow brass cylinders were spaced along the wire and fixed to it by a single grub screw. The model suspenders were made of fishing line which had been pre-stretched and treated with a beeswax coating to reduce the effect of humidity changes on its length.

The model was mounted on the horizontal turntable which fitted flush with the floor of the wind-tunnel test-chamber (see Appendix II and *Fig. 15*). Changes of the horizontal wind direction were reproduced by rotation of the turntable and the effect of vertically inclined winds was simulated by tilting the whole model about a spanwise axis near the wind-

* Unless otherwise stated, all values quoted throughout this Paper have been converted to refer to full-scale.

tunnel floor. In the latter case, for an angle of tilt α', a component of the weight of the model proportional to sin α' acted laterally, and the model normal component was reduced by the factor cos α'. In order to balance the lateral component and to restore the model normal component of force to its full value, spring-tensioned cords, inclined at an angle $\frac{\alpha'}{2}$ to the horizontal, were attached to the stiffening truss at several positions along the span. These cords were tensioned by weak helical springs and the tension adjusted until the lateral component of the weight of the platform was cancelled. In this condition the total normal component of force equalled the vertical force due to gravity for the uninclined model. The contribution of the tensioned cords to the elastic stiffnesses of the model was made very small by using long lengths of cord and springs of low stiffness.

Resonance and Damping Properties of the Model

Resonance modes and frequencies were obtained by forcing the model through a light spring by means of a reciprocating gear. With vertical bending, oscillation modes were observed showing from 0 to 6 nodes * within the centre span and corresponding to frequencies ranging from approximately 0·13 to 0·53 cycles per second. With torsional oscillations the natural frequencies ranged between 0·27 and 0·53 cycles per second for modes with from 0 to 2 nodes within the centre span. Ratios of the natural frequencies for corresponding modes of torsional and vertical oscillations were higher than was expected from considerations based solely on cable deformations and differential bending of the trusses, and showed that the influence of the torsional stiffness of the suspended structure was considerably greater than that of an actual bridge braced horizontally in one plane only. Upon investigation this high stiffness was traced mainly to the warping resistance of the model cross-girders. A correction was not attempted since a high natural-frequency ratio did not seriously detract from the usefulness of a model intended for general research purposes. The ratio was in fact more nearly representative of the bridge types incorporating two planes of horizontal bracing which were coming under consideration by the time the model was ready for test.

The logarithmic decrements of the decay of artificially excited oscillations were measured in the usual way from photographic records of amplitude. Values of about 0·05 were typical for both horizontal and vertical oscillations.

The Aerodynamic Tests

In wind tests, the tunnel speed was gradually increased from zero to the maximum corresponding on full-scale to a little over 140 miles per hour.

* In the present Paper the two nodes situated at the fixed ends of a span will be excluded in the enumeration of nodes.

The critical wind speeds and frequencies for maintained oscillations in the various modes were noted. In general the modes were observed visually but cinematograph records were taken of certain typical oscillations.*

The experiments were carried out under conditions corresponding to various vertical wind inclinations α and horizontal wind inclinations β. The effective angle α produced by a given tilting inclination α' of the model depended on the value of β, and was given to a close approximation by $\alpha = \alpha' \cos \beta$. Except for longitudinal or near longitudinal winds, the values of vertical inclination attained ranged between $\mp$ 15 degrees.

Tests were made on the influence of several design variations such as grade-line camber, tower stiffness and cable loading; and also on the effects of modifications to the external shape of the suspended structure. By covering the stiffening-truss panels with paper it was possible to simulate the aerodynamic effect of a plate-girder-stiffened bridge. In this condition the model reproduced many of the modes of oscillation which occurred on the original Tacoma Bridge.

The more important results and conclusions are given below.

(*a*) *Coupling between vertical bending and torsional motions.*—Each wind-induced oscillation observed corresponded in mode and in frequency to a natural oscillation induced by resonance tests in still air. From this experimental evidence, and from independent visual observations, it was concluded that coupling effects between the vertical bending and torsional motions had little influence on the oscillations. However, as previously noted, the natural frequency ratio $\frac{N_\theta}{N_z}$ for corresponding wave-forms was high and was more approximate to a bridge with a double rather than a single plane of lateral bracing. The effect of a closer approach to equality of the natural frequencies was investigated in the sectional-model tests to be described later.

(*b*) *Influence of oscillation wave-form.*—With a given wind inclination and model condition, all the different types of torsional oscillation appeared for approximately the same constant value of V_r. A similar conclusion applied for vertical bending oscillations. This indicates that the influence of oscillation wave-form on critical values of V_r is unimportant.

(*c*) *Influence of shape of suspended structure.*—The stability depended on the shape and arrangement of the components of the suspended structure. The model with plate-girder stiffening exhibited a high degree of instability in both vertical bending and torsional oscillations. No instability in vertical bending motion was found in any test with a truss-stiffened model. Torsional oscillations occurred for certain arrangements of the truss-stiffened model.

* Some of these records have been incorporated in a short silent film produced by the National Physical Laboratory and entitled "Oscillations of a Model Suspension Bridge in Wind."

The influence of shape is discussed in more detail in the description of the sectional-model tests.

(*d*) *Influence of elastic stiffnesses and natural frequencies.*—Critical values of V_r were not appreciably influenced by variation of the stiffnesses and natural frequencies due to structural modifications which did not involve change of shape of the suspended structure. Such modifications included variation of the tower stiffnesses, unloading the sidespan cables, and fitting a centre-tie between truss and cable.

(*e*) *Influence of wind inclination.*—The highest degree of instability was found in transverse winds ($\beta = 0$) and the stability characteristics improved progressively with increase of β. Vertical bending oscillations of the plate-girder-stiffened model persisted, but with decreasing amplitude, up to a value of β between 30 and 45 degrees; whilst weak torsional oscillations were still present at $\beta = 60$ degrees. No instability of any type was found in steady longitudinal winds.

The stability was sensitive to the vertical inclination of the wind. The highest degree of instability of the plate-girder-stiffened bridge occurred with slight negative vertical inclination, and that for the truss-stiffened bridges with slight positive inclination.

(*f*) *Effect of grade-line camber.*—The stability was not greatly influenced by variations of the grade-line camber. The indications were that a cambered grade-line yielded very slightly better stability characteristics than a level grade-line.

(*g*) *Effect of gusty winds.*—Some tests were attempted with several types of disturbed airflow, none of which, however, was necessarily representative of natural gusty winds. In longitudinal, as well as in transverse winds, irregular vertical oscillations were set up by the buffeting action of large-scale eddies shed from the gust-making device, but no tendency to torsional motion was observed.

(*h*) *Correlation between sectional-model and full-model tests.*—The full-model tests showed that critical values of V_r for specific values of the structural damping were determined by the shape of the suspended platform and were not substantially affected by other structural properties or by the wave-form of the oscillation. Also, in these tests, the highest degree of instability was produced in transverse winds. It was concluded that sectional-model tests would be adequate for stability prediction provided they yielded the same critical values of V_r as those given by the corresponding full model.

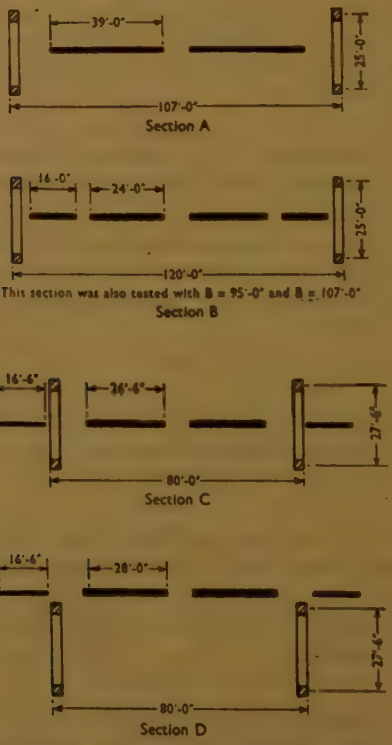
Table 1 sets out a comparison of the results obtained with the full model and with its sectional-model copy. The alignment of the vertical motion in the sectional-model tests were not strictly correct except at zero incidence, since the direction of model-motion was not altered to correspond to the incidence change. The error in alignment increased with incidence and hence may account for the lack of correlation between the results at $\alpha = \mp 10$ degrees for vertical oscillations of the plate-girder-stiffened

section. With this exception, all the results showed very good agreement between the two methods of test and support the conclusion that reliable predictions of the stability of proposed suspension bridges may be based on sectional-model tests only.

Experiments with $\frac{1}{100}$-scale Sectional Models

The models tested represented the four main types of bridge section drawn in outline in *Figs 4*; some of the actual models are illustrated by the

Figs 4



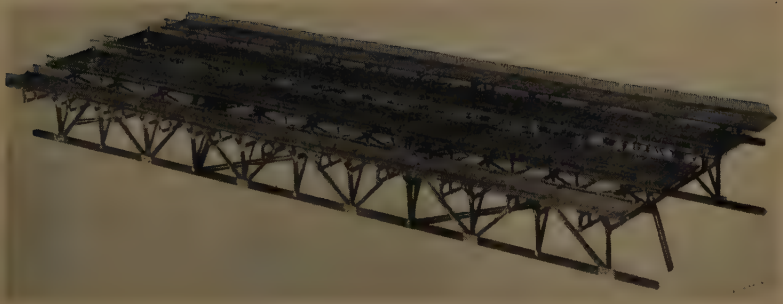
Main Types of Bridge Section Tested

photographs reproduced in *Figs 5* (facing p. 193) and *8*. They were of rigid wooden construction and represented a 340-foot length of the bridge suspended structure. Tests were made in a wind tunnel with a working section measuring 4 feet by 3·40 feet. Two types of mounting, here referred to as the " original " and the " improved," were used at different times in the investigation. These were :

Fig. 7

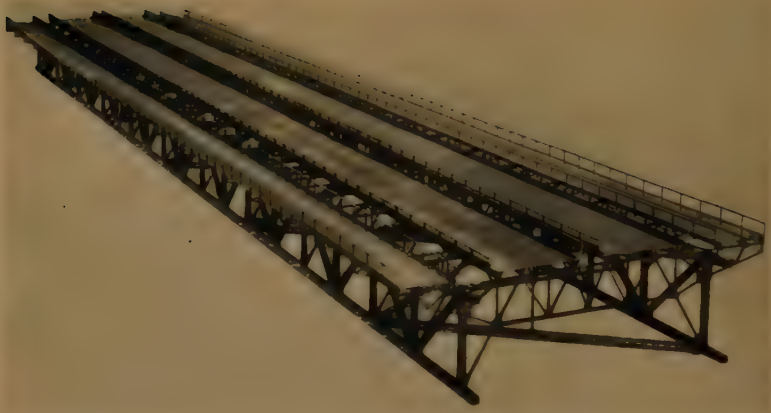
THE $\frac{1}{100}$-SCALE SECTIONAL MODEL C

Fig. 8



THE $\frac{1}{100}$-SCALE SECTIONAL MODEL D

Fig. 12



THE $\frac{1}{32}$-SCALE SECTIONAL MODEL OF THE DESIGN PROPOSED FOR THE SEVERN BRIDGE

Fig. 16

THE WIND TUNNEL FROM THE DISCHARGE END

(i) *The original mounting.*—This permitted both vertical bending and pitching motions, either singly or together. The apparatus damping was not directly variable, and the inertias of the model on this mounting were very much greater than those required by correct scaling of the prototype values. However, since in the tests $N_c = N_\theta$, the similarity conditions given on p. 199 were applicable.

The model was attached at both ends to circular plates which were supported on ball-bearings so that pitching motion could take place against the elastic constraint provided by helical springs. Each bearing and spring assembly was carried on a framework which was constrained to move vertically by a steel-strip device ; helical springs again providing stiffness. The circular end-plates fitted flush with the walls of the wind tunnel.

(ii) *The improved mounting.*—This was used for pitching-motion tests with correct inertial scaling and with the apparatus-damping variable from a low initial value. As in the case of the original mounting, the model was carried between discs. Each disc was supported on a steel-strip suspension which maintained a fixed axis of oscillation and also provided the required elastic stiffness. The damping due to this suspension was very small. Additional damping was supplied by the action of a thin segment of copper which oscillated with the model and passed between the pole-pieces of an electromagnet. The damping moment produced by the eddy currents set up in the copper was proportional to the velocity of the motion and could be readily varied by adjusting the current through the coils of the electromagnet.

Discussion of Results

The models were all stiffened by trusses of the single Warren type, but could be readily converted to represent plate-girder-stiffened sections by the attachment of solid covers to the trusses. They were usually tested with a pitching-axis placed approximately centrally with respect to the four stiffening-truss chord members but some tests were made with other axis positions. The maximum value of V_r or V_s obtainable depended on the test conditions but in all cases corresponded to full-scale wind speeds of rather more than 100 miles per hour and for most of the pitching-motion tests to rather more than 200 miles per hour. The apparatus damping of the original mounting corresponded to $\delta_{\theta s} = 0{\cdot}03$ and $\delta_{zs} = 0{\cdot}06$ approximately on the full-scale bridge. Tests were conducted at $\alpha = 15$, 10, 5, 0, -5, -10, and -15 degrees. A section is described as stable when no oscillations occurred within the experimental limits.

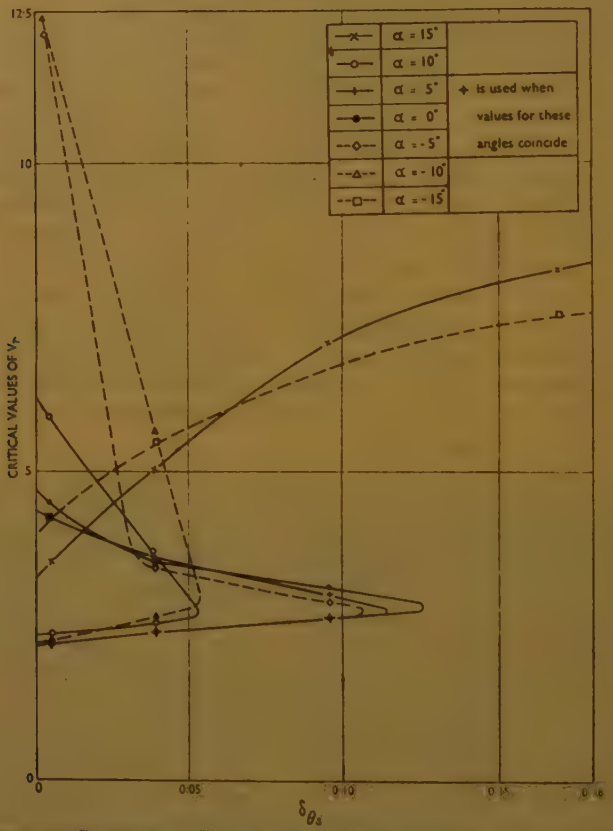
Some of the models were tested on both the original and the improved mountings. The good agreement obtained between corresponding sets of results provided experimental verification for the similarity conditions stated on p. 199 for the case where strict inertial scaling is not attempted.

The main conclusions derived from the results of the sectional-model tests are summarized below.

(*a*) *Influence of structural damping.*—Diagrams showing the influence of structural damping on stability are reproduced in *Figs 9* and *10* for the plate-girder- and truss-stiffened Sections A respectively.

Increase of $\delta_{\theta s}$ narrowed the instability range by increasing the critical

Fig. 9

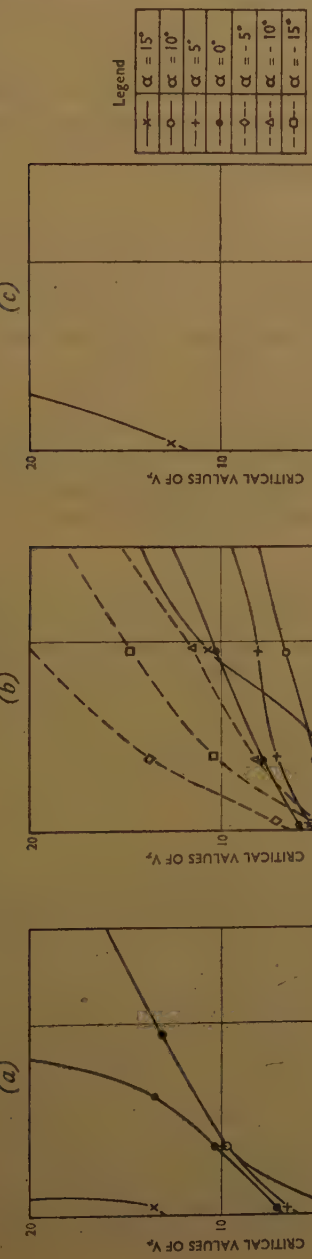


INFLUENCE OF STRUCTURAL DAMPING ON THE PITCHING OSCILLATIONS OF THE PLATE-GIRDER-STIFFENED SECTION A

speeds for the lower boundary and decreasing those for the upper boundary.* The magnitude of $\delta_{\theta s}$ necessary to prevent oscillations for all wind speeds provided a qualitative indication of the relative strengths of the instabilities.

* This was found to be generally true, although the particular examples selected for *Figs 10* did not show an upper boundary to the instability region.

(*b*) *Influence of location of pitching axis.*—The vertical position of the pitching-axis was varied in tests of the mid-deck section (B, *Figs 4*) and



Figs 10

INFLUENCE OF STRUCTURAL DAMPING ON THE PITCHING OSCILLATIONS OF THE TRUSS-STIFFENED SECTION A

of the top-deck section (D, *Figs 4*). For axes lying midway between the stiffening trusses the stability of both these sections was least when the axis was located near the level of the deck.

(*c*) *Coupling between vertical bending and pitching oscillations.*—In sectional-model tests with coupled motions, particular attention was given to ratios of the natural frequencies near unity, since it was expected that the influence of coupling would then be most marked. Except when the frequency ratio $\frac{N_\theta}{N_z}$ closely approached unity, one of the motions was always found to predominate, and the critical frequency and reduced velocity were in fair agreement with those obtained in the corresponding isolated-motion test. When $\frac{N_\theta}{N_z} = 1$ both motions were present in substantial proportions. In one instance the model was stable when its freedoms were isolated, but coupled oscillations occurred when $\frac{N_\theta}{N_z}$ was very close to unity. Hence it was concluded that sectional models could be tested satisfactorily with the vertical and pitching freedoms isolated, unless the frequencies for corresponding modes were approximately equal.

(*d*) *Influence of the form of the suspended structure*

Plate-girder sections

Plate-girder sections were in most cases represented by covering the stiffening trusses of the sections shown in *Figs 4*, and were therefore deeper than is usual in practice. Such sections were in general considerably more unstable than truss-stiffened ones (compare *Fig. 9* and *Fig. 10* (*a*)). They showed instability in both vertical and pitching motions, usually at low wind-speeds. For example, the critical values of V_r for the plate-girder-stiffened sectional model A were about 1·7 and 2·5 for vertical and pitching oscillations respectively.

Truss-stiffened sections

No vertical oscillations were excited with any truss-stiffened section tested. This result is most probably true for truss-stiffened bridges generally, since it is confirmed by model-tests elsewhere [11] and by full-scale experience.

The stability in pitching motion of the truss-stiffened sectional models depended on the form and arrangement of the structural components. It was often sensitive to the design of small structural details and especially of those, such as handrailing and roadway stringers, carried by the decks. Because of these effects, and of the numerous possible design variations, it is not considered practicable to make reliable predictions of the stability of truss-stiffened bridges by means of simple criteria only and without experiments with models. However, the results of tests with several types of bridge section and with numerous modifications to their structural

details do show certain general trends, and these may be helpful as a guide to the initial choice of section. The influence of the design and arrangement of a truss-stiffened bridge section on pitching stability was as follows :—

(i) *Stiffening trusses.*—An increase in the width/depth ratio of the stiffening-truss chords improved the stability. The tests were not sufficiently comprehensive to indicate the influence of the number or disposition of the diagonal and vertical members.

(ii) *Roadway decks*

Reservations between roadways

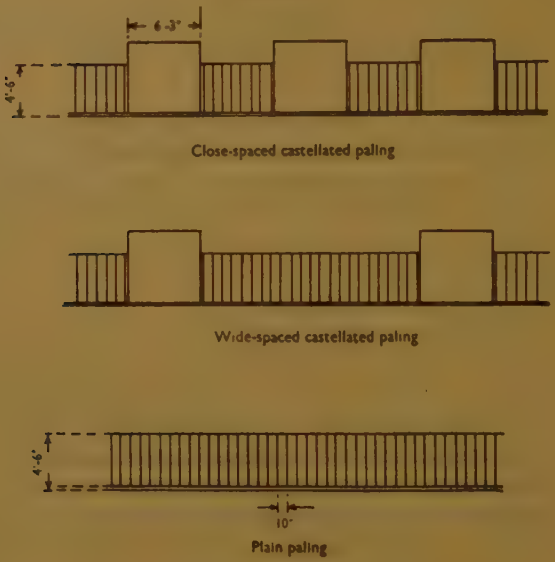
The division of the total roadway width into a number of parallel tracks, each separated by a reservation through which air could flow freely, had a very beneficial effect (compare *Figs 10* (*a*) and *10* (*b*)). Such a reservation may be formed by a grating, or may be completely uncovered and bordered by balustrades. Improvements were obtained by increasing the number and width of such reservations as, for example, in Section B which showed somewhat better stability characteristics than those of Section A.

Roadway stringers and handrailing

The structural form of the roadway stringers and handrailing had a considerable influence. Sections with plain decks (that is to say, without stringers or handrailing) showed instability at both negative and positive inclinations of the wind. Substantial improvements were effected by fitting stringers and handrailing which act as aerodynamic spoilers by shedding eddies of mixed frequencies and directions along the span. Such spoilers on the deck upper surfaces improved stability at positive angles of incidence while those on the lower surfaces were effective for negative angles. Truss-type stringers, and to a certain extent, plain paling handrailing (*Figs 11*) proved to be suitable for this purpose. The effectiveness of the paling handrailing was considerably increased by the addition of solid castellations of the form shown in *Figs 11*. (Compare the stability diagrams *Fig. 10* (*a*) and *Fig. 10* (*c*)). The use of solid-plate construction for either stringers or handrailing did not produce the same beneficial effect.

The stability was influenced by small changes of the vertical levels of the decks, both relative to each other and to the supporting structure. No indication of the deck levels most favourable to stability can be made from the few results available.

Figs 11



FORMS OF HANDRAILING

Traffic

In view of the marked influence on stability exerted by fittings to the roadway decks it was desirable to determine the aerodynamic effect of traffic. Several arrangements of models of various types of vehicular traffic were placed on the decks of Section B (see *Fig. 6*). None of these had any measurable adverse effect.

(iii) *Horizontal bracing.*—The influence of the type and position of the horizontal bracing was tested for the mid-deck Section C only, and was found to be small. The indications were that plate-girder bracing members compared unfavourably with those of open-lattice construction, and that the most favourable position for the lattice-types of bracing was just underneath the roadway decks, where the spoiling effect was most pronounced.

(iv) *Comparison of the stability of the main types of section.*.—The sections A to D shown in *Figs 4* are lettered in chronological order of test and illustrate successive stages in the evolution of a design with very good stability characteristics. A comparison between the different sections is difficult because certain small structural details influencing the stability were not common to all the types. When some of the advantageous features already indicated were included, stability was achieved with all the sections. Table 2, which lists

the stable configurations found with each type, shows that the number of stabilizing features which had to be incorporated decreased progressively through the series tested. Section A required wide stiffening-truss chords, close-spaced castellations on the hand-railing, and truss stringers. (No. 1, Table 2.) The width of the chords could be decreased (No. 2), or the extent of castellation reduced (No. 3), when the two roadways were divided into four (Section B), provided an open reservation separated adjacent roadways.* For economy in the construction of the towers and the piers the width between the trusses was reduced to 80 feet in Section C and the sidetracks were supported outside the trusses. By the use of wide chords and wide-spaced castellations on the handrailing, stability was now achieved when plate-stringers were fitted on all four roadways. (No. 6.) Castellations on the handrailing became unnecessary when truss sidetrack-stringers were used together with wide chords (No. 4) or with the narrow chords provided the lattice horizontal bracing was raised to the proximity of the deck. (No. 7.) This improvement was maintained with Section D, which had its roadway decks placed just above the top chords (Nos 8 and 9). The top-deck configuration D has the practical advantage that it permits the use of two widely separated planes of horizontal bracing which form, with the stiffening trusses, a box-structure of great torsional rigidity. The consequent increase in the natural frequencies for the torsional modes, as compared with those of a bridge with a single plane of bracing, has the twofold advantage of yielding proportionately higher critical speeds and of increasing the separation between the natural frequencies of corresponding modes in torsion and in vertical bending.

Confirmatory Tests of the Stability of the Proposed Severn Bridge

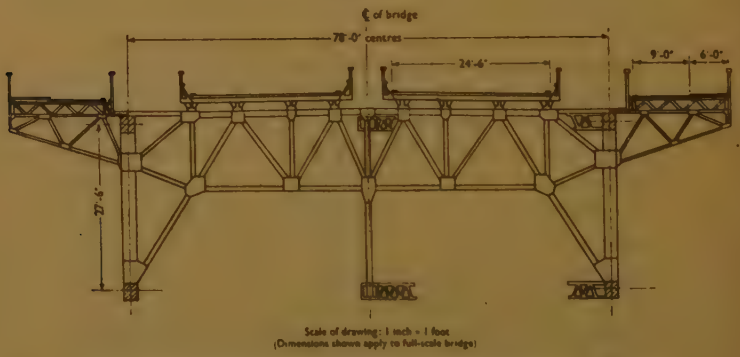
A large-scale sectional model was used to provide final confirmation of the stability of the preferred design for the Severn Bridge. An increase of the linear scale to 1/32 was considered desirable to allow more accurate reproduction of fine structural detail. Preliminary experiments had indicated that the oscillations of a still larger model might be affected by

* Each roadway of the two-roadway sectional model A represented the combination of carriageway, cycle-track, and footpath. When four roadways were used, as in sectional models B, C, and D, the two inner ones represented carriageways; the outer ones each represented the combination of cycle-track and footpath. For brevity, the outer roadways are termed "sidetracks."

the proximity of the wind-tunnel roof and floor. The tests were carried out in the large wind tunnel at Thurleigh Airfield.

The design for the proposed bridge gives a centre span of 3,240 feet with a cable-sag/span ratio of 1/11, and two sidespans each 1,000 feet long. The suspended structure is a top-deck design with stiffening trusses 27 feet 6 inches deep and spaced 78 feet apart. Its general form will be clear from the sectional drawing of *Fig. 13* and from *Fig. 12* (facing p. 205), which shows a photograph of the model structure. High torsional rigidity is ensured by the two planes of horizontal bracing at the levels of the top and bottom stiffening-truss chords. Each sidetrack comprises a cycle-track and footpath 9 feet and 6 feet wide respectively. The sidetrack balustrades are made up of a top horizontal railing supported at intervals on vertical

Fig. 13



Cross-section Showing General Arrangement of Model

posts, the area between the posts being filled with panels of 3/4-inch diameter vertical bars spaced at 6-inch centres. The 24-foot-wide carriageways are separated by an open reservation and are bordered by balustrades similar to those of the sidetracks except that a 6-inch-by-3-inch channel is incorporated at approximately mid-height as a crash barrier and that, up to this level, the bars slope outwards over the reservations. Both sidetrack and carriageway decks are supported on plate-web-girder stringers.

The model represented a 600-foot length of the suspended structure. Considerable care was taken in its construction to reproduce all the important features of the full-scale design. The only significant departure from this was in the use of solid members for the model stiffening trusses. The full-scale design shows stiffening-truss chords of approximately square section built up on three sides with solid plates and with lacing on the underneath side. The diagonal and vertical members of the trusses are of rectangular section formed by solid plates and two opposing sides and lacing on the remaining sides. Since the solid surfaces of all these members would have been bluff to the transverse winds in which the tests

were made it was considered that a solid representation was adequate. The model was made so that it could be separated into two equal lengths, each corresponding to 300 feet. Tests were carried out with the full length and with the half length, to provide information on the influence of the model-length/width ratio.

The model was mounted on steel spring suspensions carried by frames which tilted to give the desired vertical incidence to the wind. The suspension arrangements permitted pitching combined with either vertical translation or lateral translation motions, as well as tests with each of these motions isolated. Tests with the lateral freedom included were necessary, since with the top-deck structure the lateral and pitching motions were inertially coupled, owing to the offset of the centre of mass above the elastic axis of the structure. In all three motions viscous-oil dampers were provided to enable the amount of structural damping represented to be varied. The position of the pitching-axis of rotation could be altered. The three positions used in the tests were midway between the stiffening trusses at the levels of the top, bottom, and midway between the top and bottom chords.

The inertias of the model conformed with the requirements for strict inertial scaling. Initially the frequencies were adjusted to values 3·2 times greater than those calculated for full-scale. This yielded a speed-scale of 1/10 and enabled the tests to be carried out up to wind speeds corresponding to approximately 140 miles per hour. The minimum values of $\delta_{\theta s}$ and δ_{zs} were 0·01 and 0·06 respectively. No instability of any type occurred in these tests which covered an incidence range of between ∓ 15 degrees.

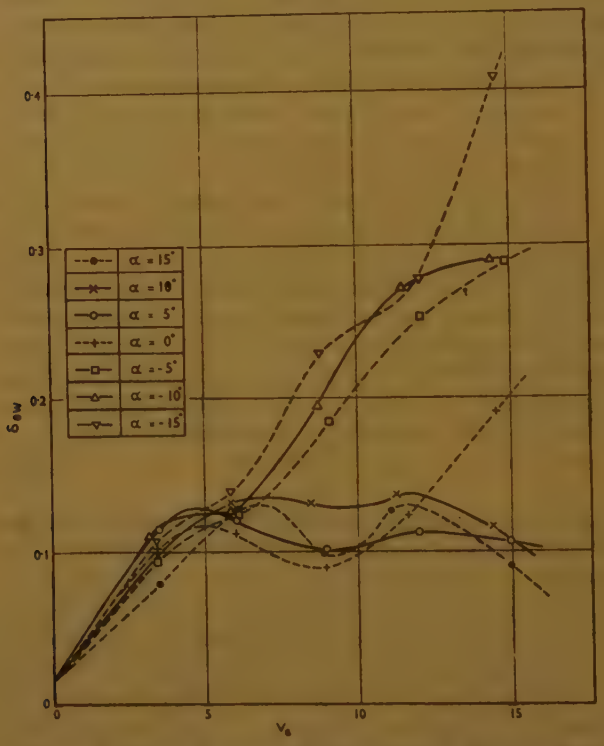
No further tests involving the lateral and vertical motions were made. With the more important pitching motion, N_θ was reduced to allow tests to be carried out up to wind speeds equivalent on full-scale to more than 200 miles per hour. The value of $\delta_{\theta s}$ for these tests was 0·02. No instability was observed for these conditions, but to provide further information the variation of the damping rate due to wind only ($\delta_{\theta w}$) with wind speed was measured. The curves of $\delta_{\theta w}$ against V_s for several wind incidences are reproduced in *Fig. 14*. These show that for negative wind incidences the damping rate became increasingly positive. For positive incidences the damping rate increased with the initial increases of V_s and thereafter maintained a substantial positive value for the whole speed range tested. These results verified that the design of suspended structure proposed for the Severn Bridge was satisfactory from the standpoint of aerodynamic stability.

In addition, the model was used to confirm some of the results found on the $\frac{1}{100}$-scale models and also to obtain some information on the effect of the width/length ratio of sectional models. Some unstable configurations of the model (for example, that obtained by covering the central reservation) were tested with model lengths representing 600 feet and

300 feet. The results obtained with these two model lengths showed only small differences.

One of the configurations of the $\frac{1}{100}$-scale model Section D (*Fig. 8*) represented the proposed design for the Severn Bridge but with less accuracy in the construction and in the reproduction of detail than was possible with the $\frac{1}{32}$-scale model. With $\delta_{\theta s} = 0{\cdot}02$, instability of the smaller scale model in pitching motion was found in downwinds of 15 degrees

Fig. 14



Damping Rates of Pitching Oscillations Due to Wind
($\frac{1}{32}$-scale model of proposed Severn Bridge)

inclination at a critical speed corresponding to about 160 miles per hour. The more favourable result found with the $\frac{1}{32}$-scale model may have been due to the increase of linear scale, or more probably, to the small differences in shapes of the two models.

Acknowledgements

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The present Paper is published by permission of the Director of the Laboratory and with the approval of the Ministry of Transport. During the investigation much direct help was received from the Ministry of Transport, and especially from the staff responsible for the design, construction and maintenance of the wind-tunnel structure at Thurleigh Airfield. Facilities for carrying out the experiments at that airfield were courteously granted by the Ministry of Supply, and many valuable services were rendered by the Ministry of Works.

The work would not have been possible without the advice and close collaboration of the staff of the Joint Engineers for the Severn Bridge (Messrs Mott, Hay and Anderson and Messrs Freeman, Fox and Partners). Great benefit was also derived throughout the research from arrangements made with investigators of the parallel problem in the U.S.A. for an unrestricted exchange of information.

Dr R. A. Frazer, F.R.S., was in charge of the investigation. His many valuable comments and suggestions contributed much to the experimental side of the work, and he developed the methods which were used for the calculation of the stiffnesses and natural frequencies of the bridge structures. The Author is also indebted to other colleagues for their collaboration and advice, and for their able assistance in the construction and erection of the models and in the conduct of the experimental work.

The Report is accompanied by nine photographs, seven sheets of diagrams, from which the half-tone page plates and the Figures in the text have been prepared, and by the following four Appendices.

APPENDIX I

Note on the Damping Properties of Bridges

The total damping rate (δ) of a structure in still air is made up of the contributions due to the structure only (δ_s) and that due to the surrounding air (δ_A). The application of model-test results to prototype prediction requires a knowledge of the values of δ_s for both model and prototype. There is as yet no reliable method for calculating these values for a proposed bridge, and measurements of δ on actual bridges, which might be used for statistical estimates, have only been made on bridges of short-span.[16] The values of δ found for short-span bridges varied from 0·05 to more than 0·2. Values for models are, of course, readily obtained by decaying-oscillation experiments.

The aerodynamic part of the total damping arises from the effects of viscosity and pressure. For pitching oscillations of similar sectional models, dimensional analysis requires :—

$$\delta_{\theta A} = \frac{\rho B^4}{2I_\theta} f\left[\frac{\nu}{B^2 N_\theta}, \theta_0\right] \quad \ldots\ldots\ldots\ldots \quad (1)$$

where θ_0 denotes the amplitude of oscillation.

Both theory and experiment suggest that the relation (1) will take the form :

$$\delta_{\theta A} = \frac{\rho B^4}{2I_\theta}\left[a_0 + a_1\theta_0 + a_2\theta_0^2 + \ldots\right] \quad \ldots\ldots\ldots \quad (2)$$

in which a_0, a, etc., are coefficients dependent only on the parameter $\nu/B^2N\theta$.

The equivalent expression for vertical translatory motions is :—

$$\delta_{zA} = \frac{\rho B^2}{2I_z}\left[b_0 + b_1\left(\frac{z_0}{B}\right) + b_2\left(\frac{z_0}{B}\right)^2 + \ldots\right] \quad (3)$$

where z_0 denotes the amplitude.

The theoretical evidence on this subject is exceedingly scanty. The aerodynamic forces on a circular cylinder of diameter B_1, oscillating in still air with frequency N_z at right angles to its own length and with a *small amplitude*, have been calculated by Stokes.[17] For small values of the parameter $\nu/B_1^2N_z$, his analysis leads to the approximation :

$$b_0 = 2\pi^{\frac{1}{2}}\sqrt{\frac{\nu}{B_1^2 N_z}} \quad (4)$$

If it is supposed that the values of δ_{zA} for small oscillations of the bridge section of the type proposed for the Severn Bridge do not differ very widely from those given by a circumscribing cylinder, the data (expressed in lb.-foot-second units) appropriate to the lowest frequency of oscillation on full scale may be taken as :—

$$B_1 = 117\,;\ N_z = 0{\cdot}13\,;\ \rho = 0{\cdot}0766\,;\ \nu = 1{\cdot}56 \times 10^{-4}\,;\ I_z = 1{\cdot}41 \times 10^4$$

Formula (3), with the coefficient b_0 only included, then gives :—

$$\delta_{zA} = 1{\cdot}23 \times 10^{-4}$$

This suggests that the aerodynamic damping δ_{zA} for *small-amplitude* vertical oscillations of full-size bridges may be very small in comparison with the structural damping δ_{zs}.

Some values of $\delta_{\theta A}$ were measured during the course of the Severn Bridge investigation. The results for the 1/100-scale models A and D gave respectively the relations :—

$$\delta_{\theta A} = \rho B^4/2I_\theta\,(0{\cdot}01 + 0{\cdot}29\,\theta_0)$$

$$\delta_{\theta A} = \rho B^4/2I_\theta\,(0{\cdot}05 + 3{\cdot}40\,\theta_0)$$

Measurements of δ_{zA} were not made, but the following values, for Section A and D respectively, were estimated from the measured values of $\delta_{\theta A}$, assuming the damping forces to be uniformly distributed across the width,

$$\delta_{zA} = \rho B^2/2I_z\,(0{\cdot}17 + 21{\cdot}9\,z_0/B)$$

$$\delta_{zA} = \rho B^2/2I_z\,(0{\cdot}27 + 32{\cdot}2\,z_0/B)$$

When the terms containing z_0/B are neglected, these expressions yield values of 0·0053 and 0·0045 for δ_{zA}. These compare with theoretical values of 0·00355 and 0·00386 given by equation (4) for the 1/100-scale models on which the measurements were made.

Values of δ_A for complete bridges or their models will be influenced by the oscillation wave-form. They may be calculated by a method suggested by Vincent and McHugh,* provided the sectional-model values and the oscillation wave-form are known. For pitching oscillations the method yields :—

$$\delta_{\theta A}(\text{full model}) = \frac{\rho B^4}{2I_\theta}\left[a_0 + a_1\theta_0\frac{\int f(x)^3\,dx}{\int f(x)^2\,dx} + a_2\theta_0^2\frac{\int f(x)^4\,dx}{\int f(x)^2\,dx} + \ldots\right] \quad (5)$$

where x denotes the distance along the span,

$f(x)$ expresses the wave-form with respect to a convenient reference point,

and θ_0 denotes the amplitude at the reference point.

APPENDIX II

The Large Wind Tunnel

Design and Construction †

The wind tunnel (see *Figs 15* and *16*, facing p. 205) was not required as a permanent structure and hence the main considerations governing its design were that of low cost of

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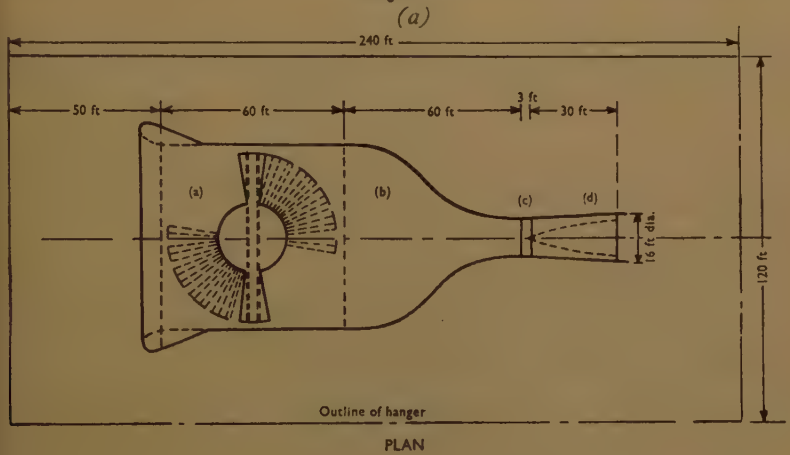
* It is understood that this method will be published in Part V of Ref. 11.

† The Bridge Section of the Ministry of Transport was responsible for the structural design of the wind tunnel, and the construction was carried out by the Direct Labour Organization, Eastern Division of the Ministry of Transport.

construction rather than that of high aerodynamic efficiency. For this reason the tunnel was of the non-return-flow type and used only one fan. It was erected in a disused aircraft-hangar and was raised from the hangar floor only by the few feet necessary to accommodate a well for the turntable. The fabric of the tunnel consisted mainly of a timber framework lined with wallboard, and the roof was suspended from Bailey bridge girders supported at their ends by vertical concrete pillars. The four main components of the tunnel were :—

(i) *The test chamber.*—[(a) in *Figs 15.*] This had a floor area of 60 feet by 60 feet and was 7 feet high. The air entered the test chamber through a conventional bell-mouth fairing and a wire-mesh screen for smoothing the airflow. In the centre of the

Figs 15



THE WIND TUNNEL

chamber was a 55-foot-long turntable contained within a shallow quadrantal pit which allowed a rotation of 90 degrees. The flooring over the central part of the turntable was carried by the turntable itself, and the outer annular area was filled by wedges of 5-degree angles constructed of boarded trestles, which were lifted successively when the model had to be rotated relative to the wind direction.

(ii) *The contraction chamber.*—[(b) in *Figs 15.*] The contraction chamber was 60 feet long and in this distance the cross-section developed smoothly from a 60-feet-by-7-feet rectangle to the 12-foot-diameter circle of the adjoining fan annulus. A wire-mesh screen at the front of the contraction chamber helped to ensure even airflow in the test chamber.

(iii) *The fan and fan annulus.*—[(c) in *Figs 15.*] The fan annulus, 12 feet in diameter and 3 feet in length, was of all-timber construction and stiffened to ensure that the small clearance between it and the two-bladed fan was maintained. The fan was driven by a concentric 130-horse-power motor with fine speed control.

(iv) *The diffuser.*—[(d) in *Figs 15.*] This was of circular section, 30 feet in length, and expanded from the fan annulus to a diameter of 16 feet at the discharge.

Performance

Tests of the aerodynamic characteristics were made initially on a 1/12-scale model of the tunnel and hangar.

In the actual wind tunnel, extensive measurements of the distribution of airflow within the test chamber showed that at all speeds up to the maximum of 22 feet per second the velocity variations both along the length of the chamber and vertically were less than 3 per cent. In the horizontal direction across the test chamber, a variation of windspeed of nearly 10 per cent was recorded at a distance of 5 feet from the sides. A variation of this order was predicted by the model tunnel tests and was not considered important in view of the clearance of nearly 5 feet between the full-model anchorages and the sides of the chamber.

APPENDIX III

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Detailed results of the investigation are given in the following unpublished progress reports issued by the National Physical Laboratory. Copies may be seen in the Institution Library.

Ref. No.	*Title*
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2. NPL/Aero/150, 152, 160, 162 and 177.	Wind Tunnel Tests on Sectional Bridge Models. Interim Reports Nos 1 to 5.
3. NPL/Aero/185 and 199 .	Experiments on the Aerodynamic Stability of Suspension Bridges. Results of tests of a full model.
4. NPL/Aero/201 . . .	The Aerodynamic Stability of Suspension Bridges: Tests of a 1/100-scale sectional model of a top deck type of suspended platform.
5. NPL/Aero/213 . . .	The Aerodynamic Stability of Suspension Bridges. Comparison of results obtained with a full model and its sectional model copy.
6. NPL/Aero/218 . . .	The Aerodynamic Stability of Suspension Bridges: Tests of a 1/32-scale sectional model of the design finally proposed for the Severn Bridge.
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APPENDIX IV

Conversion Factors

(*British to Metric Units*)

The following list shows the factor by which the British value should be *multiplied* in order to obtain the corresponding metric value.

1 inch	= 2·54 centimetres
1 foot	= 0·3048 metre
1 square inch	= 6·4516 square centimetres
1 ton	= 1,016·0 kilogrammes
	= 1·016 metric ton (tonne)
1 mile per hour	= 1·6093 kilometre per hour

TABLE 1.—COMPARATIVE RESULTS FROM SECTIONAL-MODEL AND FULL-MODEL TESTS

The models represented a mid-deck bridge of Section A, *Fig. 4*. In its standard condition, the model decks were separated by an open reservation and were fitted with paling-type handrailing and truss-type roadway stringers. The values for the structural damping were :—

Full Model— $\delta_{zs} = 0{\cdot}035$, $\delta_{\theta s} = 0{\cdot}05$

Sectional Model— $\delta_{zs} = 0{\cdot}06$, $\delta_{\theta s} = 0{\cdot}05$

The full model was tested up to speeds corresponding to values of V_r of 15 and 8 for vertical and torsional oscillations respectively. Higher values were reached in the sectional-model tests, but for the purpose of this comparison the above values are taken as the limits for both types of test and a result is given as *stable* when no oscillations occurred up to these limits. Only the lowest critical values are quoted here since on the full model the critical speeds for the upper limit of the instability range were usually marked by the onset of a further mode of oscillation.

[*Table 1 is continued on p. 221.*]

TABLE 1—*continued*

Model configuration	α degrees	Lowest critical values of V_r			
		Vertical oscillations		Torsional oscillations	
		Sectional model	Full model	Sectional model	Full model
Standard	— 15, — 10, — 5, 0, 5 10, 15			Stable	Stable
Standard but with all handrailing removed.	10			7·0	6·8
	— 15, — 10 —5, 0 5, 15			Stable	Stable
Standard but with solid-plate handrailing.	0			3·4	3·8
	— 15, —10 — 5, 5 10, 15	Stable	Stable	Stable	Stable
Standard but with close-spaced castellated-paling handrailing.	— 15, — 10 — 5, 0, 5 10, 15			Stable	Stable
Standard but with the handrailing on the inner edges of the carriageways removed and a solid cover fitted over the central reservation.	15			5·0	4·7
	10			5·4	4·8
	5			7·2	6·9
	0, — 5 — 10, — 15			Stable	Stable
Standard but with the stiffening trusses covered to represent the aerodynamic effect of solid-plate-girder stiffening.	15	Stable	Stable	5·5	Stable
	10	Stable	1·8	2·6	2·5
	5	1·7	0·7	2·4	2·7
	0	1·8	1·6	2·4	2·6
	— 5	1·8	1·7	2·4	2·5
	— 10	Stable	1·7	2·8	2·4
	— 15	Stable	Stable	5·8	4·2

TABLE 2.—STABLE CONFIGURATIONS OF BRIDGE SECTIONS A, B, C, AND D. (*Figs 4.*)

Within the experimental limits no instability in pitching motion was found with any of the tabulated model configurations. The tests were conducted up to wind speeds corresponding on full-scale to about 200 miles per hour, at wind incidences of 15, 10, 5, 0, − 5, − 10, − 15 degrees. The structural damping corresponded to approximately $\delta_{gs} = 0{\cdot}03$.

In all the configurations listed, the reservations between adjacent roadways were uncovered.

No.	Section	Width/depth ratio of chords of stiffening trusses	Type of handrailing (see *Fig. 11*)	Type of deck stringers		Type and position of horizontal bracing	
				Sidetracks	Carriageways	Type	Level †
——→*		High	Close-spaced castellated	Truss	Truss	Latticed box	Near deck level.
1	A	2·2	Close-spaced castellated	Truss	Truss	Plate girder	Bottom chords.
2	B	1·0	Wide-spaced castellated	Truss	Truss	Plate girder	Bottom chords.
3		2·2	Plain paling	,,	,,	,, ,,	,, ,,
4	C	2·2	Plain paling	Truss	Plate	Plate girder	Bottom chords.
5		1·0	Wide-spaced castellated	,,	,,	,, ,,	,, ,,
6		2·2	,, ,,	Plate	,,	,, ,,	,, ,,
7		1·0	Plain paling	,,	,,	Latticed box	Underneath decks.
8	D	1·0	Plain paling	Truss	Plate	Latticed box	Top chords.
9		1·0	,, ,,	,,	,,	,, ,,	Top and bottom chords.
10	D ‡	1·0	Plain paling	Plate	Plate	Latticed box	Top and bottom chords.

* The qualities which tended to promote stability are given in this row

† The chords referred to in this column are those of the stiffening trusses.

‡ This result was obtained with the 1/32-scale model of the design proposed for the Severn Bridge; the remainder with 1/100-scale sectional models.

Paper No. 5848

"A New Method of Gauging Stream Flow with Little Afflux by Means of a Submerged Weir of Triangular Profile"

by

Edwin Samuel Crump, C.I.E.

(Ordered by the Council to be published with written discussion) †

SYNOPSIS

The high cost of single-gauge measuring-weirs, resulting mainly from the large afflux required to ensure modularity, is often found to be prohibitive. Substantially to reduce afflux and cost, a measuring-weir must be capable of operating when submerged (that is, in the non-modular range) and this involves double-gauging with its attendant complexities and liability to error. A new method of double-gauging applied to a weir of triangular profile largely overcomes these defects. The upstream gauge records the total head T by means of a Pitot pier ; the downstream gauge records the pressure-head P at a point on the downstream face of the weir a short distance from the crest. Tests made by the Author on a model-weir of standard profile show that the modular coefficient of discharge is sensibly constant, and that the ratio $f = Q/Q_M$ of actual to modular discharge (for a given value of T) is a unique function of the ratio P/T of the two gauge-readings. A table presenting this unique relationship provides a ready means of calculating the discharge from recorded values of T and P.

Introduction

In countries like India, where large revenues are derived from vast irrigation schemes, experience has shown the wisdom of encouraging the installation of devices of the highest attainable degree of precision for the measurement and distribution of irrigation water. With ample funds available, the device commonly applied for this purpose is some form of the long-throated weir, in which, when properly designed, the discharge is given by the gauge-reading observed or recorded at a single station situated upstream of the weir. The method is simple and the discharge can be ascertained with less than 1 per cent error. Unfortunately the cost is high and, in countries where water has little earning value, would generally be regarded as prohibitive. In Britain, for example, it would seem that cost is a primary consideration, and that the prevalent demand is for a weir of much lower cost, capable of measuring the discharge of a stream over the whole range of flow, even though this can only be done with a

† Correspondence on this Paper should be received at the Institution by the 1st July, 1952, and will be published in Part I of the Proceedings. Contributions should be limited to about 600 words.—Sec. I.C.E.

degree of accuracy much inferior to that of a long-throated weir. The only sure way of reducing cost is to sacrifice modularity, and to design the weir to operate in the submerged, or non-modular, range of performance.

In this range, since the discharge is now a function of two independent variables instead of one, the measurement of discharge is a much more intricate process. The standard procedure is, first, to establish the value of the modular coefficient C_M from which to calculate the modular discharge corresponding to the upstream head T; and then to ascertain the manner in which the reduction factor $f = Q/Q_M$ varies with different degrees of submergence, that is to say, with the ratio of the downstream to the upstream gauge-reading. Difficulty in obtaining reliable results from double-gauging arises from three main causes :—

(1) Abnormal turbulence due to power-dissipation causes the surface of the stream to be disturbed for a considerable distance downstream of the weir. A downstream gauge purporting to record tail-water surface-level must therefore be placed far enough downstream to be immune from these disturbances, but because of the frictional loss of head between weir and gauging station, the gauge then fails to give a true measure of the downstream head in the vicinity of the weir.

(2) In the majority of cases, the modular coefficient C_M is not constant, but increases with the upstream head T. Unless known from previous experience, the relationship between C_M and T must be determined by laboratory tests on a model of the proposed weir.

(3) If C_M is not constant, the relationship between the reduction factor f and the ratio of submergence is not unique, but is different for different values of T. This relationship must in any case be determined by model-tests, so that if the relationship is not unique, the number of tests required is greatly increased, and the labour of computing discharge, from observed or recorded gauge-readings, becomes extremely tedious.

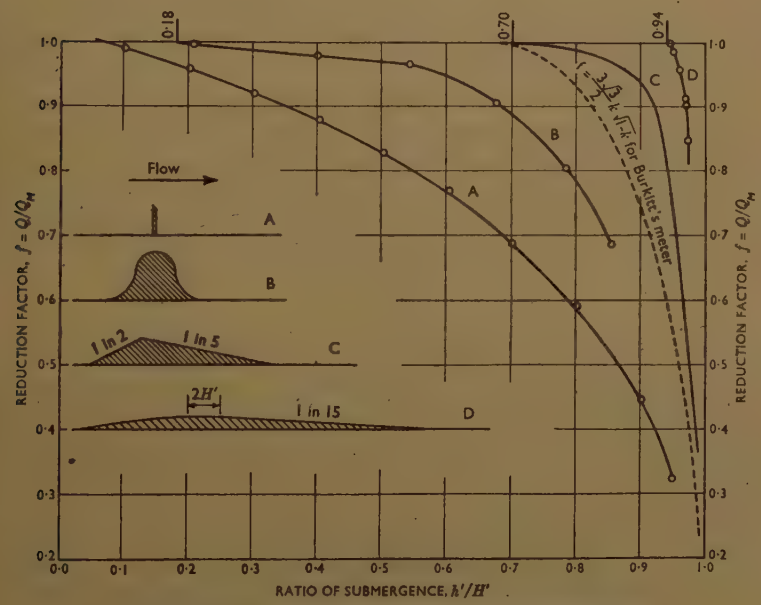
In seeking for a form of weir likely to prove suitable for double-gauging, it was realized that the best, and possibly the only, chance of success lay in finding a weir which avoided these difficulties and complexities. It was found that in the case of all weirs which are rounded or have a flat facet in the vicinity of the crest, C_M increases with T, whereas all weirs of triangular profile investigated by Bazin exhibited, for each particular profile, an approximately constant value of C_M.

Constancy of coefficient simply implies that the flow pattern, in the vicinity of the " control section," conforms to the Froude scale of similitude : the pattern for one head T_1 is a scalar facsimile of the pattern for another head T_2. Moreover, such similitude also applies to all linear quantities associated with the pattern, and it is of particular interest and importance to note that it must apply to the shape and pressure-head of the low-pressure pocket of eddying water which, in all weirs of triangular profile, is formed just downstream of the crest of the weir, where the effective jet is deflected upwards by the upstream face, before descending

to become adherent to the downstream face some distance from the crest; so that, at the control section where the underside of the jet reaches the highest point of its trajectory, the jet overlies a pocket or roller of circulating water which makes no contribution to the effective flow.

This consideration suggested an alternative method of double-gauging. Instead of a remote gauging-station recording, with indifferent accuracy, the downstream head h', the pressure-head P of the low-pressure pocket could be recorded by a piezometer connecting with the pocket through tappings in the downstream face of the weir. Preliminary tests made on a small model of the triangular weir (shown as Model C in *Fig. 1*) showed

Fig. 1



SUBMERGENCE CURVES FOR WEIRS OF VARIOUS PROFILES

that, when operating in the modular range, the pressure-head P was always about 25 per cent of the upstream head T; whereas, at the modular limit, the downstream head h' was about 75 per cent of T. This meant that in the non-modular range the gauge P would be about 3 times as sensitive as the gauge h', thereby offering greater accuracy in measuring the discharge.

Although Bazin's investigations were confined to the modular range and therefore promised similitude in that range only, there appeared to be no reason why the principle of similitude should not also apply in the non-modular range. If it did so apply, then, with different values of T, but

with the same value of the ratio P/T, discharges would be similar, that is to say, proportional to $T^{\frac{3}{2}}$; so that the reduction-factor f would be a unique function of the submergence ratio P/T. If these hopes were realized in a model-test, it would mean not only that the three difficulties mentioned above had been overcome; but that the value of C_M, and the relationship of f to P/T, given by the model-tests, could confidently be applied to a full-scale weir of similar profile. The extent to which these hopes were realized will be apparent from the test results recorded later in the Paper.

A further advantage to be gained by piezometric gauging is that the two gauge-wells or float-chambers recording the values of T and P can be placed so close together that the two recording mechanisms can be driven from a single clock; and values of T and P can, if desired, be recorded on a single chart, thereby ensuring synchronization of the two variables, and greatly facilitating the process of computing the discharge from a standard curve or Table.

In the sections which follow, various matters relative to the behaviour and gauging of measuring-weirs are discussed; examples are given to illustrate and compare the widely different behaviour of weirs of different contour; the convenience—in the case of submerged meters—of direct measurement of the upstream head, by means of a Pitot-pier, is stressed; and an interesting example is given of an alternative method of double-gauging applied to the case of a long-throated weir.

Critical Flow and the Control-Section

Whilst the principle of "minimum energy," as applied to determine the coefficient of a long-throated weir, is well known, it is perhaps not commonly realized that the principle is of general applicability. The principle postulates the existence of a "bottle-neck" or control section, and asserts that, at this section of the effective nappe, the variables, in terms of which both the discharge Q and the total head T can be expressed, will assume the unique values required to give both the differentials dQ and dT a zero value. For example, if the discharge is constant, so that $dQ = 0$, the second condition $dT = 0$ implies that T will have the minimum value. Alternatively, if T is constant, so that $dT = 0$—as would be the case where the weir formed the outlet from a large reservoir—then $dQ = 0$ implies that for this value of T the discharge will have the maximum value. It is seen that the two alternative conceptions of maximum discharge for a given head, or of minimum head for a given discharge, both lead to the same mathematical results (obtained by partial differentiation of Q and T) and yield two independent relationships between the variables.

If, for example, the variables are:—

x to denote depth of effective nappe,

p to denote manometric pressure at underside of nappe,

and r to denote radius of curvature of underside of nappe,
then

$$Q = F(x, p, r) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and one of the conditions to be satisfied is :

$$dQ = \frac{\partial F}{\partial x}\,dx + \frac{\partial F}{\partial p}\,dp + \frac{\partial F}{\partial r}\,dr = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the weir has a rounded crest, so that the nappe at the control section is adherent to a solid boundary, then r is constant and the last term in (2) disappears. If, on the other hand, the profile of the weir is so abrupt that separation occurs, the control section is situated a little downstream of the crest at the section where the underside of the nappe attains its highest level. If the nappe is freely aerated, and tail-water low enough, it may ride on a pocket of air at atmospheric pressure ; if the weir is drowned, the nappe rides on a pocket of water where pressure is determined by the level of the tail-water. In either case, the pressure p is constant, and the middle term of equation (2) disappears.

In general, it may be stated that if the functional relationship of Q and T to the variables could be expressed mathematically, the discharge of any weir, free or drowned, could be determined by applying the principle of critical flow.

Modularity

When the discharge of a weir depends only upon the upstream head, and is independent of the downstream head, that is to say, of tail-water level, its performance is said to be modular. When, with an increase of downstream head, this independence ceases and the discharge depends upon both upstream and downstream head, the performance is said to be non-modular.

In cases where the nappe leaving a weir falls freely into air, the performance is manifestly modular. In other less obvious cases, modularity is frequently associated with the formation, downstream of the weir, of a standing-wave which effectually protects the control section from the effect of relatively high pressures prevailing in the deep water downstream of the wave. The matter may be illustrated by considering the behaviour of the triangular weir shown as Model C in *Fig. 1*, this being the profile that the Author proposes to adopt as a standard in connexion with his new method of double-gauging. With this profile, separation takes place at the angular crest where the nappe is deflected upwards by the upstream face, rides on a pocket of eddying water that makes no contribution to the flow, and—if the tail-water is low enough—returns to become adherent to the downstream face at some distance from the crest. If the standing-wave is remote, conditions of flow at the control section depend upon the pressure prevailing in the eddying pocket underlying it. The limit of

modularity is attained when the wave-front reaches the downstream end of the pocket, so that the higher pressure of the buffer is communicated to the pocket. At this phase, the pocket is inflated, the curvature and velocities of the nappe are reduced, and the discharge-coefficient of the weir falls below its modular value.

Gauging

In the modular range of performance, a single gauging-station, recording the upstream head, suffices to determine the discharge at all stages of flow.

In the non-modular range, the discharge can only be ascertained by introducing a second gauging-station downstream of the weir. The discharge then depends upon two independent variables in a manner which, if not known from previous experience, must be determined by experiment. This being the case, it is important to choose these variables so that the functional relationship between them and the discharge Q is independent of the local peculiarities of the two gauging stations, and dependent only upon the profile of the weir. If, for example, a gauge-well or float-chamber fitted with a clock-driven recording mechanism is installed at each of the two gauging stations where the heights H and h of surface above crest-level are recorded, the correct procedure is to adopt as variables :

$$H' = H + \frac{V^2}{2g} \text{ and } h' = h + \frac{v^2}{2g}$$

where A and a denote the cross-sectional areas ; $V = Q/A$ and $v = Q/a$ denote the mean velocities at the two stations. The functional relationship between Q, H' and h' can then be expressed as :

$$Q = f \cdot Q_M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Q_M is the modular discharge corresponding to the upstream head H', and f is a function—peculiar to the type of profile adopted—of the ratio of submergence (h'/H').

The method of double-gauging visualized above is subject to the following disconcerting considerations :—

(1) Whilst the upstream station would be close to the weir, the downstream station must be placed far enough downstream to be immune from the disturbing effects of turbulence and surface oscillation occasioned by the dissipation of $\frac{Q(H' - h')}{8{\cdot}8}$ horse-power downstream of the weir. To meet this requirement, the distance between the two stations may be great enough for disregard of frictional losses between the two stations to result in an appreciable under-estimate of the downstream head h'. And, as will be seen later, a small error or uncertainty in h' may involve a large error in the reduction-factor f.

(2) With separate clocks at the two stations, defective synchronization may give a false indication regarding simultaneous values of H' and h'.

(3) If the cross-sections of the channel at the gauging stations are liable to alteration by deposition or scour of bed-material, frequent soundings would be necessary to furnish correct values for A and a.

(4) Since H' and h' contain terms involving Q, neither Q_M nor f can be expressed as explicit functions of observed quantities; so that, in calculating the value of Q, recourse must be had to a tedious "trial and error" method.

If the weir is two-dimensional, that is to say, has the same profile throughout the width B between parallel vertical side-walls, the modular discharge is given by the equation:

$$Q_M = C_M . B . (H')^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where the modular coefficient C_M may be sensibly constant, or may increase with H'. Whilst much data is available regarding the values of C_M for weirs of various profile, so little is known regarding the relationship between f and (h'/H') that in designing a double-gauge measuring-weir, it would almost certainly be necessary to determine the relationship by preliminary model-tests.

Typical Submergence-Curves

The behaviour of a weir is most readily presented by a submergence-curve showing corresponding values of the reduction factor $f = Q/Q_M$ and the ratio of submergence (h'/H'). Such curves, showing the results of tests made on four different two-dimensional weirs, are shown in *Fig. 1.* Curve A is plotted from results obtained by J. B. Francis and Ftely-Stearns for a sharp-crested weir; the other three curves show results obtained, at various times, by the Author. A point to be noted is the very wide range in values of the modular limit. These range from a negative value (not shown) for the sharp-crested weir of Model A, to the very high value of 94 per cent for the long-throated weir of Model D.

The apparent anomaly presented by Curve A in rising above the line $f = 1{\cdot}00$ calls for explanation. In this case the modular limit is reached well before tail-water rises to crest-level. Before the limit is reached, the nappe is discharging into free air, the air-pocket under the nappe being maintained at atmospheric pressure by a generous supply of air from vents in the side walls. Below the air-pocket is a mass of eddying water, the surface of which stands at a higher level than the tail-water; so that, before tail-water rises to crest-level, the air-pocket is replaced by water at less than atmospheric pressure, the effect of which is to increase the curvature and velocities of the nappe at the control section. As a result of this change, the first effect of submergence is to cause an abrupt increase in the discharge-coefficient. With increasing submergence, pressure on the underside of the nappe increases, thereby reducing the coefficient until it regains its original modular value at a drowning ratio of about 7 per cent.

In Model B, the cylindrical crest was formed by a drum, rotated to investigate pressures. When modular, the nappe adhered to the drum, the pressure at the crest being less than atmosphere. The modular limit was reached when conditions at the control section were altered at a submergence ratio of about 18 per cent. The kink in the curve at about 54 per cent submergence marks the point where the nappe abruptly changed from " plunging " to " floating."

Model C is the Author's standard weir for double-gauging. Its selection was prompted by results obtained by Bazin for a variety of weirs of triangular profile. A merit common to all triangular weirs is that with a varying head H' the modular coefficient C_M remains virtually constant. The submergence curve for Model C is plotted from test-results presented later in the Paper. It is seen that the model has a high modular limit of 70 per cent, and that with a submergence ratio of 80 per cent the departure from modularity is less than 1 per cent.

Tests on Model D were carried out by the Author in 1918. The model used was an idealized replica of what was, at that time, the standard design of weir for river barrages at the headworks of perennial irrigation canals in the Punjab. It may be regarded as the forerunner of the standard measuring weirs subsequently brought into general use on irrigation canals in India.

In considering the utility and significance of submergence-curves, the following points may be noted :—

(1) The afflux caused by the weir being given by the expression $H' - h' = (1 - h'/H') \, . \, H'$, its value expressed as a multiple of H' can be seen from the curve at a glance, since the multiplier is merely the complement of the submergence-ratio.

(2) Since flow ceases when h' and H' have the same value, it is clear that every submergence-curve must terminate at the point $(h'/H') = 1{\cdot}0$; $f = 0$.

(3) The higher the value of the modular limit, the more steeply does the curve descend. This implies that, with double-gauging of h' and H', errors in the value of f, arising from the uncertainty that always attaches to h', increase with the modular limit. For instance : the flattest part of Curve D, occurring immediately after the modular limit is exceeded, is as steep—and therefore liable to as great error—as the steepest part of Curve A, where non-modularity has so far advanced that the discharge Q is only 40 per cent of Q_M.

The inescapable conclusion to be drawn from the curves of *Fig. 1* is that : if accuracy is the only consideration, the best form of weir for this method of double-gauging is the sharp-crested weir of Model A, or some equally abrupt and afflux-producing alternative.

A disadvantage of weirs of abrupt profile is, however, that the large afflux they produce necessitates the provision, downstream of the weir, of

costly flooring to protect the bed against erosive action resulting from high power-dissipation.

The Long-Throated Weir

Curve D of *Fig. 1* suggests that a long-throated weir of high modularity is unsuitable in cases where double-gauging is contemplated; the curve of descent is everywhere so steep that little reliance could be placed on results obtained in the non-modular range.

Confined to use in the modular range, its merits, as an accurate single-gauge measuring device, are outstanding. With a proper choice of the width B, it can be designed to operate, in the modular range, over the whole range of flow without causing unduly high afflux at any stage. To ensure modularity, however, an ample margin of safety must be allowed to cover uncertainty in regard to the stage-discharge characteristics of the natural channel, and also to guard against any possibility of the modular limit being exceeded by changes in the regime of the downstream channel. The afflux allowed under these precautions would usually involve, especially during floods, the dissipation in a standing-wave of a large amount of power downstream of the weir—namely, $Q(H'-h')/8{\cdot}8$ horsepower—rendering it necessary to protect the structure of the weir by means of a suitable stilling-basin. The length of a well-designed long-throated weir might well be as much as twelve times the maximum depth of flow of the natural channel. In Britain, its high cost would in most cases be regarded as prohibitive.

Pitot Gauging

The basic purpose of the upstream gauge is, in all cases, to ascertain the total head obtaining at the control section of the weir. With this object in view, the head $H' = H + V^2/2g$ is usually ascertained by recording the height H at a station so close to the weir that the frictional losses occurring between the station and the control section are small enough to be disregarded. To avoid uncertainty regarding the sectional area A from which the mean velocity $V = Q/A$ is calculated, it is always advisable to introduce at the station a level floor high enough always to remain clear of deposited material. If this is done, then, in the modular range only, the relationship between Q_M and H is unique and can be determined once for all, so that the chart-paper of the automatic recorder can be graduated to read discharge directly. In all other cases, that is to say, if the gauging section is liable to deposition or scour, or if the weir is non-modular, the discharge Q can only be found by trial and error. In such cases there is a definite advantage in registering and recording the value of the upstream head H' directly, by means of a Pitot tube, or, more practically, by means of a Pitot pier embodied in the design of the weir. Where this is done, the

upstream head so measured is, in the Paper, designated by the letter T to indicate its direct recording.

Burkitt's "Headless" Meter

In a long-throated measuring-weir, the position of the control section is approximately half-way down the throat. Although double gauging is only necessary if the weir goes out of modularity, it is always advisable to provide for this eventuality by constructing a gauge-well communicating with the mid-point of the throat, and registering the depth of flow x at the control section. It would take the place of the usual downstream gauging-station and render the latter unnecessary.

Such an arrangement, intended for use in the non-modular range only, was introduced on Punjab canals by F. H. Burkitt who described the device as a "headless" meter. The term "headless" was used, first, to emphasize the absence of a standing-wave and its associated loss of head, and secondly, because in Burkitt's design the overall loss of head was reduced to a minimum by a gentle divergence downstream of the throat.

In this device, with T and x recorded, the discharge is given by the equation:

$$Q = Bx\sqrt{2g(T-x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or putting $x = kT$

$$Q = k\sqrt{1-k}\,\sqrt{2g}\,B\,.\,T^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Differentiating for the maximum gives $k = \frac{2}{3}$ and

$$Q_M = \frac{2}{3\sqrt{3}}\sqrt{2g}\,.\,B\,.\,T^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

so that

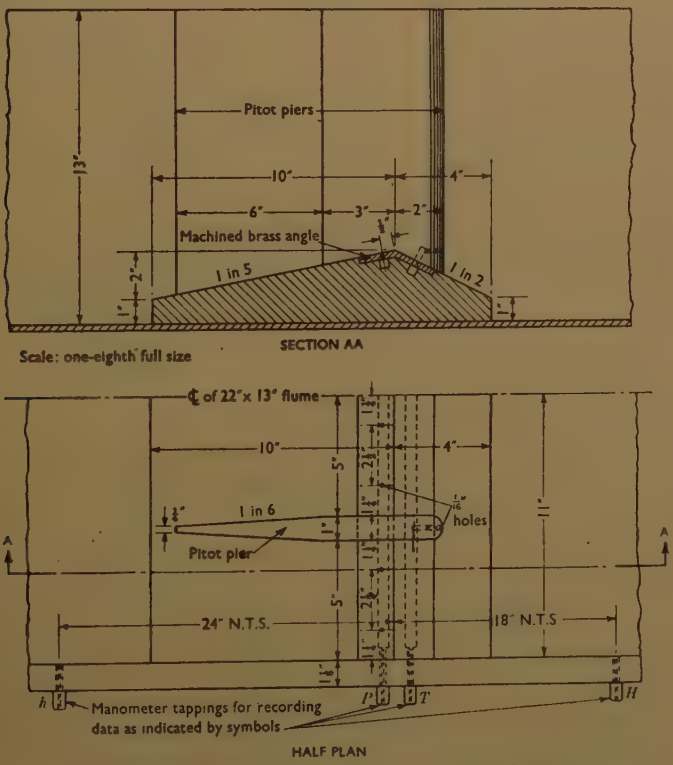
$$f = Q/Q_M = \frac{3\sqrt{3}}{2}\,.\,k\sqrt{1-k} \quad . \quad . \quad . \quad . \quad (8)$$

Regarding the ratio $k = x/T$ as an alternative to the ratio h'/H', the submergence-curve given by equation (8) is shown by the dotted line of *Fig. 1.* Comparison with Curve D—applicable to the usual method of double-gauging—shows that the latter is about five times steeper than the dotted curve. Error in assessing the value of the reduction-factor f from the ratio of the two observed or recorded gauge-readings is therefore five times greater for the usual method of double-gauging than for Burkitt's method. An additional advantage of the latter method is that it avoids the defects discussed under the heading "Modularity," and it should be adopted, in preference to the usual method, whenever it is intended to use a long-throated weir in the non-modular range and double-gauging becomes an essential requirement in the measurement of flow.

The Model

The model-weir used by the Author in obtaining the results tabulated below is shown in *Figs 2*. It was fitted into a rectangular flume 22 inches wide and 13 inches deep with its crest approximately 3 inches above the steel floor of the flume. The weir-block had an upstream face at an inclination of 1 in 2, and a downstream face at an inclination of 1 in 5, giving a crest-angle of 142 degrees 7 minutes. The block was truncated to give

Figs 2



Test Model of Standard Weir of Triangular Profile

an overall length of 14 inches with two vertical faces: one, 4 inches upstream of the crest; the other, 10 inches downstream of the crest. The machined brass angle-plate, forming the crest of the weir, had eight small holes of $\frac{1}{16}$-inch diameter, drilled on a line $\frac{1}{2}$ inch from the crest, communicating with an underlying duct. Two Pitot-piers 1 inch wide divided the effective width of the weir into one central bay of 10 inches,

and two side bays each of 5 inches width. In the nose of each pier, at crest level, a single hole of $\frac{1}{16}$-inch diameter led to a second duct connected to the T-manometer. Four manometers, recording the values of h, P, T and H, respectively, were mounted side by side on a vertical board carrying a scale with its zero at crest-level. Discharges were measured by means of a standard V-notch situated upstream of the model. In each of the six runs recorded below, the discharge Q was kept constant while the tail-water level, recorded by the h-manometer, was varied by means of an undershot regulating gate situated at the downstream end of the flume. For each setting of the gate, simultaneous readings of the four manometers were taken.

Test Results

The following notation is used throughout the Tables :—

Q denotes discharge in cusecs.
F ,, height of crest above upstream floor in inches.
h ,, ,, tail-water surface above crest in inches.
P ,, ,, piezometric column above crest in inches.
T ,, ,, Pitot-gauge column above crest in inches.
H ,, ,, head-water surface above crest in inches.
$h' = h + 12v^2/2g$ = downstream head in inches, where :
$v = 144Q/22{\cdot}0\ (h + 2{\cdot}807)$ feet per second.
$H' = H + 12V^2/2g$ = upstream head in inches, where :
$V = 144Q/22{\cdot}0\ (H + F)$ in feet per second.
P/T = effective drowning ratio expressed in terms of P and T.
h'/H' = drowning ratio expressed in terms of h and H.
$C_T = 24{\cdot}94\ Q/T^{\frac{3}{2}}$ = coefficient of weir in terms of T.
$C_{H'} = 24{\cdot}94\ Q/(H')^{\frac{3}{2}}$ = coefficient of weir in terms of H'.

For the test of Table 5 only, the upstream bed was raised by gravel-filling to make $F = 1{\cdot}50$ inch ; for all other tests, F had a value of 3·063 inches.

In the following Tables, and in *Figs 3* and *4*, values of weir-coefficients are expressed in British units in which the discharge Q (in cusecs) is given by :—

$$Q = C_T \,.\, B \,.\, T^{\frac{3}{2}}$$

where the width B and head T are expressed in feet.

Thus, for an observed head of T inches, the effective width of the model weir being 20 inches, the value of C_T is given by :—

$$C_T = Q \div \left(\frac{20}{12}\right) \left(\frac{T}{12}\right)^{\frac{3}{2}} = 24{\cdot}94\ Q/T^{\frac{3}{2}}$$

and similarly for values of $C_{H'}$.

TABLE 1

$Q = 0{\cdot}5769$ cusecs $F = 3{\cdot}063$ inches

No.	h	P	T	H	P/T	C_T	h'	H'	h'/H'	C_H'	Remarks
1	9·197	9·203	9·228	9·245	0·9973	0·513	9·216	9·262	0·9950	0·510	
2	8·980	8·963	9·005	9·004	0·9954	0·532	8·999	9·022	0·9974	0·531	
3	8·495	8·489	8·540	8·512	0·9941	0·577	8·516	8·532	0·9982	0·577	
4	5·902	5·831	5·965	5·923	0·9774	0·987	5·946	5·956	0·9983	0·990	
5	3·195	2·876	3·420	3·357	0·8410	2·275	3·269	3·422	0·9555	2·273	
6	3·431	3·225	3·640	3·607	0·8861	2·072	3·500	2·667	0·9544	2·049	
7	2·803	2·333	3·095	3·058	0·7537	2·642	2·887	3·129	0·9225	2·599	
8	2·324	1·247	2·753	2·682	0·4529	3·150	2·426	2·762	0·8782	3·134	
9	1·882	0·961	2·668	2·594	0·3603	3·301	2·003	2·678	0·7480	3·284	
10	1·522	0·850	2·644	2·582	0·3217	3·347	1·670	2·665	0·6266	3·307	Modular.

TABLE 2

$Q = 0{\cdot}7929$ cusecs $F = 3{\cdot}063$ inches

No.	h	P	T	H	P/T	C_T	h'	H'	h'/H'	C_H'	Remarks
1	9·414	9·376	9·487	9·463	0·9883	0·684	9·448	9·496	0·9951	0·676	Floating nappe.
2	9·036	9·001	9·105	9·093	0·9885	0·728	9·073	9·128	0·9940	0·717	,, ,,
3	8·620	8·572	8·685	8·664	0·9870	0·782	8·659	8·703	0·9949	0·770	,, ,,
4	5·669	5·544	5·811	5·769	0·9540	1·428	5·741	5·835	0·9838	1·403	,, ,,
5	4·131	3·833	4·386	4·331	0·8738	2·177	4·238	4·425	0·9578	2·125	,, ,,
6	3·424	2·861	3·800	3·728	0·7527	2·700	3·557	3·840	0·9263	2·629	,, ,,
7	3·066	2·043	3·489	3·410	0·5856	3·070	3·215	3·533	0·9101	2·987	,, ,,
8	2·576	1·270	3·305	3·222	0·3842	3·328	2·754	3·352	0·8215	3·223	Plunging nappe.
9	0·928	0·958	3·238	3·142	0·2959	3·433	1·297	3·276	0·3960	3·335	Modular : standing wave remote.

TABLE 3

$Q = 1{\cdot}1720$ cusecs $\quad F = 3{\cdot}063$ inches

No.	h	P	T	H	P/T	C_T	h'	H'	h'/H'	C_H'	Remarks
1	7·270	7·150	7·475	7·375	0·9565	1·430	7·378	7·476	0·9869	1·430	
2	6·970	6·840	7·180	7·090	0·9526	1·519	7·085	7·196	0·9846	1·514	
3	6·695	6·575	6·920	6·820	0·9501	1·606	6·817	6·933	0·9833	1·601	
4	6·430	6·245	6·660	6·570	0·9377	1·701	6·558	6·688	0·9806	1·690	
5	6·140	5·960	6·410	6·290	0·9298	1·801	6·277	6·415	0·9785	1·799	
6	5·845	5·655	6·120	5·990	0·9240	1·931	6·020	6·124	0·9830	1·929	
7	5·440	5·205	5·760	5·665	0·9036	2·115	5·602	5·809	0·9644	2·088	
8	5·090	4·790	5·480	5·360	0·8741	2·279	5·266	5·515	0·9549	2·257	
9	4·770	4·365	5·220	5·080	0·8361	2·451	4·961	5·245	0·9458	2·433	
10	4·460	3·860	4·940	4·775	0·7814	2·662	4·669	4·954	0·9425	2·651	
11	4·150	3·280	4·660	4·490	0·7038	2·906	4·377	4·682	0·9349	2·885	
12	4·020	2·960	4·560	4·400	0·6491	3·002	4·256	4·597	0·9258	2·966	
13	3·875	2·590	4·450	4·275	0·5820	3·114	4·122	4·479	0·9203	3·084	
14	3·645	2·070	4·295	4·120	0·4819	3·284	3·910	4·333	0·9024	3·241	
15	3·520	1·885	4·265	4·075	0·4420	3·319	3·795	4·290	0·8846	3·290	
16	3·380	1·735	4·230	4·060	0·4102	3·360	3·668	4·276	0·8578	3·306	
17	3·205	1·560	4·180	3·990	0·3732	3·420	3·510	4·211	0·8336	3·383	
18	− 1·250	1·075	4·065	3·890	0·2645	3·566	− 0·582	4·117	− 0·1414	3·499	Wave remote
19	0·450	1·070	4·070	3·880	0·2629	3·560	1·489	4·108	0·3625	3·511	Wave at R.D. 18″
20	0·920	1·075	4·070	3·885	0·2642	3·560	1·712	4·113	0·4162	3·504	,, ,, 12″
21	1·480	1·075	4·075	3·880	0·2638	3·553	2·078	4·108	0·5059	3·511	,, ,, 10″ } Modular.
22	1·780	1·075	4·090	3·885	0·2629	3·534	2·303	4·113	0·5599	3·504	,, ,, 8″
23	2·075	1·110	4·090	3·880	0·2714	3·534	2·537	4·108	0·6176	3·511	,, ,, 6″
24	2·420	1·220	4·100	3·885	0·2976	3·521	2·823	4·133	0·6864	3·504	,, ,, 5″
25	2·760	1·360	4·130	3·885	0·3293	3·483	3·115	4·112	0·7575	3·506	,, ,, 4″
26	3·150	1·920	4·250	3·880	0·4518	3·336	3·460	4·107	0·8425	3·512	,, ,, 3″
27	4·200	3·420	4·760	4·395	0·7185	2·815	4·424	4·592	0·9634	2·971	Floating and undulating nappe.

Table 4

$Q = 1{\cdot}4371$ cusecs $F = 3{\cdot}063$ inches

No.	h	P	T	H	P/T	C_T	h'	H'	h'/H'	C_H'	Remarks
1	8·860	8·750	9·090	8·960	0·9626	1·308	8·981	9·074	0·9898	1·311	Floating nappe.
2	8·400	8·270	8·660	8·540	0·9550	1·406	8·531	8·662	0·9849	1·406	,, ,,
3	8·000	7·830	8·260	8·130	0·9479	1·509	8·141	8·262	0·9853	1·509	,, ,,
4	7·580	7·380	7·855	7·760	0·9395	1·628	7·733	7·901	0·9787	1·614	,, ,,
5	7·000	6·775	7·320	7·205	0·9255	1·810	7·171	7·361	0·9742	1·795	,, ,,
6	6·445	6·140	6·720	6·690	0·9137	2·057	6·638	6·863	0·9672	1·993	,, ,,
7	5·990	5·660	6·445	6·280	0·8782	2·190	6·203	6·468	0·9590	2·179	,, ,,
8	5·460	4·920	5·930	5·780	0·8297	2·484	5·701	5·991	0·9516	2·444	,, ,,
9	5·110	4·450	5·660	5·490	0·7862	2·662	5·373	5·715	0·9401	2·623	,, ,,
10	4·790	3·850	5·390	5·200	0·7143	2·864	5·076	5·441	0·9329	2·824	,, ,,
11	4·445	2·855	5·055	4·840	0·5648	3·154	4·759	5·104	0·9324	3·108	,, ,,
12	4·070	2·200	4·880	4·665	0·4508	3·324	4·419	4·940	0·8945	3·264	,, ,,
13	3·910	2·010	4·820	4·605	0·4170	3·387	4·276	4·885	0·8753	3·320	,, ,,
14	3·770	1·840	4·770	4·565	0·3857	3·440	4·152	4·848	0·8565	3·358	Plunging nappe.
15	3·410	1·540	4·710	4·500	0·3270	3·506	3·837	4·788	0·8014	3·421	Wave at R.D. 5″
16	3·275	1·430	4·690	4·480	0·3049	3·529	3·722	4·770	0·7803	3·440	,, ,, 6″
17	3·135	1·370	4·680	4·475	0·2927	3·540	3·603	4·765	0·7561	3·446	,, ,, 6½″
18	2·940	1·310	4·660	4·460	0·2811	3·563	3·440	4·751	0·7241	3·461	,, ,, 7″ Modular.
19	2·740	1·265	4·660	4·460	0·2715	3·563	3·277	4·751	0·6898	3·461	,, ,, 8″
20	2·460	1·240	4·650	4·440	0·2667	3·574	3·056	4·733	0·6457	3·481	,, ,, 9″
21	2·150	1·220	4·650	4·440	0·2624	3·574	2·823	4·733	0·5965	3·481	,, ,, 10″

TABLE 5

$Q = 1{\cdot}4371$ cusecs $F = 1{\cdot}500$ inches

No.	h	P	T	H	P/T	C_T	k'	H'	k'/H'	C_H'	Remarks
1	8·581	8·387	8·739	8·617	0·9593	1·387	8·708	8·617	0·9921	1·378	Floating nappe.
2	7·115	6·881	7·451	7·253	0·9234	1·762	7·282	7·253	0·9752	1·756	,, ,,
3	6·280	5·911	6·678	6·444	0·8845	2·074	6·480	6·444	0·9665	2·064	,, ,,
4	5·337	4·720	5·872	5·587	0·8038	2·519	5·586	5·587	0·9444	2·491	Nappe begins to plunge.
5	4·926	3·980	5·544	5·184	0·7179	2·745	5·202	5·184	0·9371	2·741	,, ,, ,,
6	4·391	2·730	5·106	4·688	0·5346	3·107	4·709	4·688	0·9200	3·094	Plunging nappe.
7	3·353	1·597	4·788	4·326	0·3336	3·421	3·789	4·812	0·7874	3·396	Wave at R.D. 5″: not yet modular.
8	1·985	1·362	4·731	4·258	0·2876	3·477	2·703	4·755	0·5685	3·456	Wave at R.D. 10″: modular.

For this test only, the bed upstream of the weir was raised by gravel filling to within $1\frac{1}{2}$ inch of weir-crest level thereby increasing the velocity of approach by nearly 25 per cent compared with the previous case, where the upstream bed was 3·063 inches below crest-level.

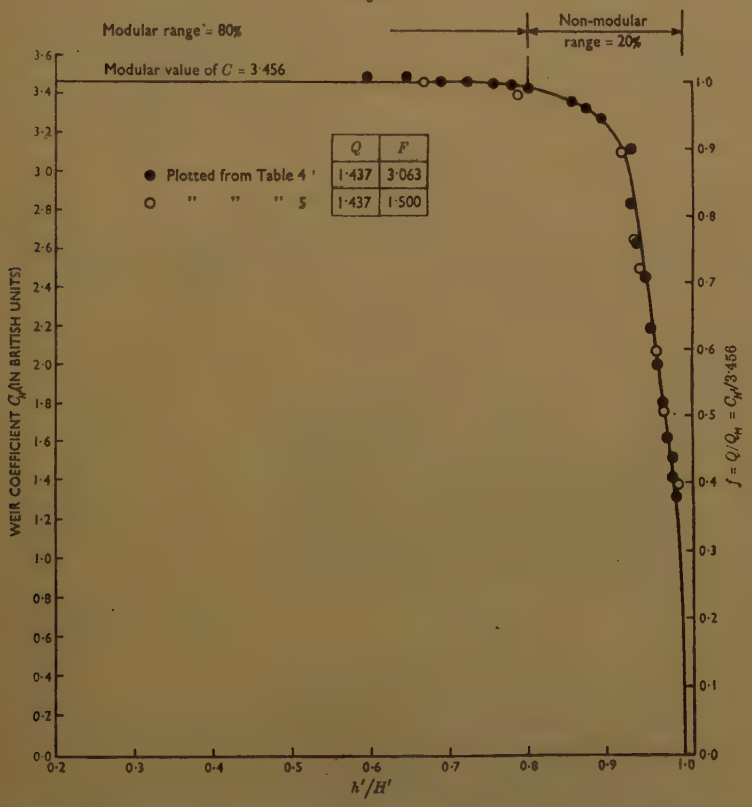
TABLE 6

$Q = 1{\cdot}510$ cusecs $F = 3{\cdot}063$ inches

No.	h	P	T	H	P/T	C_T	k'	H'	k'/H'	C_H'	Remarks
1	1·850	1·260	4·790	4·405	0·2630	3·592	2·691	4·731	0·5688	3·660	Wave Remote } Definitely
2	2·665	1·295	4·790	4·395	0·2704	3·592	3·274	4·723	0·6948	3·667	Wave at R.D. 9″ } modular.
3	2·920	1·375	4·790	4·390	0·2871	3·592	3·478	4·718	0·7372	3·675	,, ,, 7″
4	3·660	2·105	4·940	4·390	0·4261	3·430	4·096	4·718	0·8682	3·675	,, ,, $4\frac{1}{2}$″. Just not modular.
5	4·385	3·140	5·220	4·390	0·6015	3·158	4·738	4·718	1·0040	3·675	Curl at R.D. $5\frac{1}{2}$″ } Floating and
6	4·840	3·780	5·445	5·210	0·6942	2·964	5·152	5·476	0·9408	3·220	,, ,, 11″ } undulating nappe.

In *Fig. 3* corresponding values of $C_{H'}$ and h'/H' taken from Tables 4 and 5 are shown by small circles. A curve drawn fairly through these points shows that, subject to a tolerance of 1 per cent, the standard weir has a modular coefficient $(C_{H'})_M$ of 3·456, and a modular range $(h'/H')_M$ of about 80 per cent. It is of special interest to note that the performance,

Fig. 3



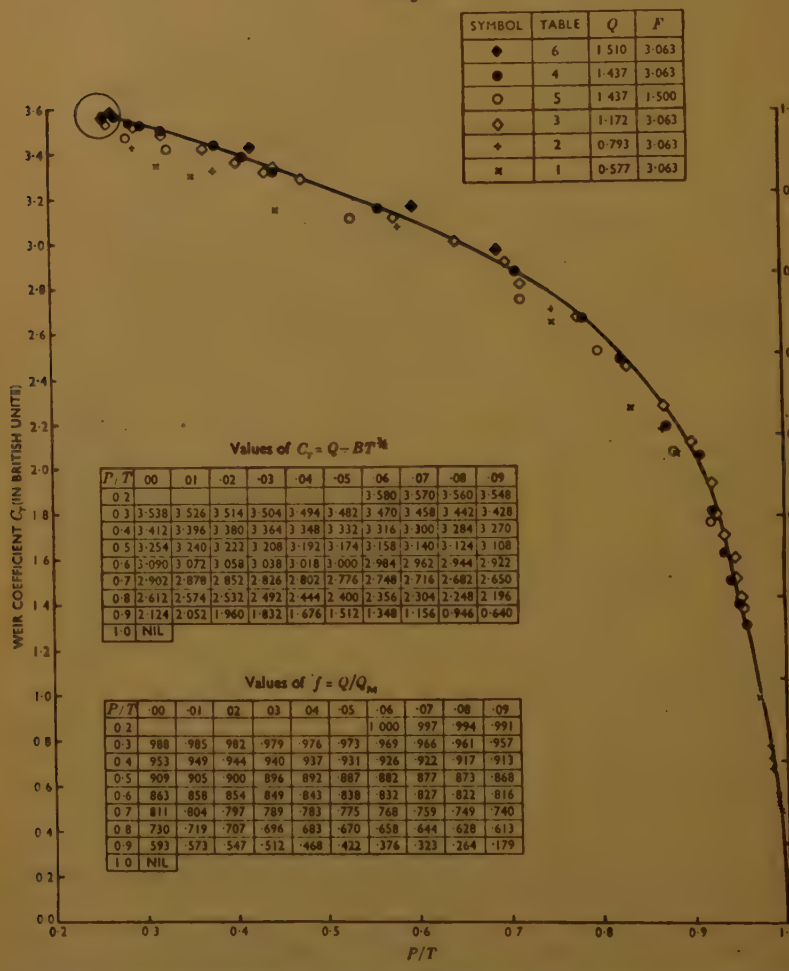
DISCHARGE CHARACTERISTICS OF STANDARD WEIR OF TRIANGULAR PROFILE, EXPRESSED IN TERMS OF RATIO h'/H'

as represented by the curve, is apparently unaffected by a heavy deposit of bed material at the upstream gauging station.

In *Fig. 4* corresponding values of C_T and P/T have been plotted, a separate legend being assigned to each of the six Tables. Disregarding results for the two lower discharges, where the scale-effect to be expected with a small model is apparent, a smooth curve has been drawn through the numerous results of Table 4. Allowing for scatter due to experimental error, all results for the three higher discharges conform to the

curve within $\pm 2\frac{1}{2}$ per cent. Subject to an error of this magnitude the results confirm the existence of a unique relationship between the reduction factor f and the submergence ratio F/T. For a standard weir, B feet

Fig. 4



DISCHARGE CHARACTERISTICS OF STANDARD WEIR OF TRIANGULAR PROFILE, EXPRESSED IN TERMS OF RATIO P/T

wide, having the same profile as the Author's model, the modular discharge Q_M is given by the formula :

$$Q_M = 3{\cdot}580\, BT^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

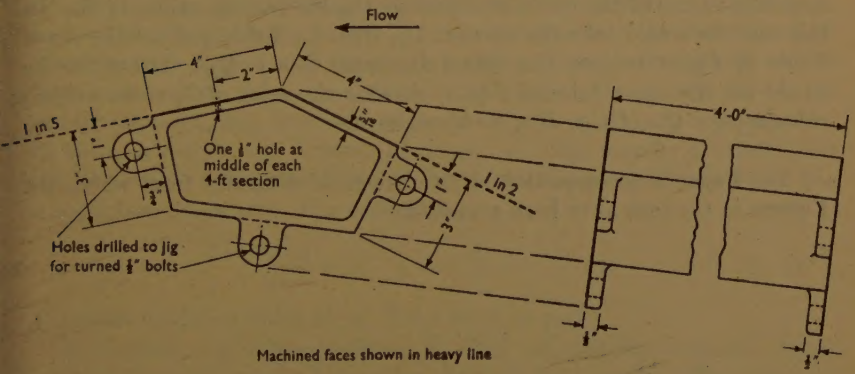
and the values of f required to give the actual discharge $Q = f \cdot Q_M$ are as tabulated in *Fig. 4.*

These values have been read from the submergence curve recommended for adoption as a standard. The curve starts from the point $f = 1{\cdot}0$; $(P/T) = 0{\cdot}26$, indicated on the diagram by a large circle. It will be noticed that, in the absence of a tail-water gauge, this method of piezometric gauging gives no information regarding the actual level of the tail-water. Knowledge of this level is quite superfluous, since a ratio of 0·26 between the two gauge readings P and T connotes modular performance, and the single terminal point of the curve covers the whole of the modular range regardless of tail-water level.

Practical Design

In practice, in designing a measuring-weir for double-gauging by Pitot and piezometer, in order to keep afflux low, the height F of the weir above

Figs 5



Combined Duct and Crest-Angle

the bed of the natural channel should not be more than one-quarter of the maximum depth of flow in the natural channel at the site of the weir. The maximum upstream head would then be about $3F$. The two faces of the weir should be carried down to bed level to give the weir block an overall length of $7F$. Vertical side-walls, high enough to contain the largest flow anticipated, should run parallel for the length of the weir block, and be carried into the banks with upstream and downstream extensions curved to a radius of $6F$. The Pitot pier should be of the same height as the side-walls. It should have a semi-circular nose; run parallel from $1{\cdot}0F$ upstream to $1{\cdot}5F$ downstream of the crest; and terminate at the

end of the weir block in a fish-tail $3{\cdot}5F$ long. A single Pitot orifice of $\frac{1}{4}$-inch bore should be at crest-level. It should be formed in a metal mouthpiece filed truly flush with the nose of the pier and might communicate with the float-chamber through a 4-inch pipe embedded in the weir block. To ensure accuracy and durability the crest or apex of the weir must be defined by an embedded section of non-corroding metal such as gunmetal.

An actual example of such a section, in which a gunmetal casting forms a combined crest-angle and duct, is shown in *Figs 5*. The complete crest is made up of 4-foot lengths bolted together with fitted bolts to ensure alignment. Each section is machined on the upper and end-faces, and has a central hole of $\frac{1}{8}$-inch bore drilled 2 inches from the crest.

Evaluation of Discharge

In practice, simultaneous values of the upstream-head T and the pressure-head P would both be recorded in feet. From these two values, the engineer concerned would evaluate the corresponding discharge Q at such epochs as he might consider desirable. He might find it convenient to draw up his own table, calculated from formula (9) above, showing corresponding values of the modular discharge Q_M for various values of T. In this case, he would take the value of Q_M from his Table and use the lower Table of *Fig. 4* to give the actual discharge $Q = f \cdot Q_M$. Otherwise he would use the upper table of *Fig. 4*, showing the value of C_T to be used in calculating: $Q = C_T \times B \times T^{\frac{3}{2}}$ by slide-rule.

The Paper is accompanied by five sheets of diagrams from which the Figures in the text have been prepared.

Paper No. 5840.

"Maximum Condition, Approximation, and Superposition in Theories of Plasticity and of Earth-Pressure"

by

Professor Hermann Craemer, Dr Ing.

(*Ordered by the Council to be published in Abstract form*) †

The theory of plasticity, in its most common form, assumes that the normal stresses, σ, remain constant once the yield stress, σ_y, has been reached. In problems dealing with limiting states of soil, the angle between the resultant stress and the perpendicular to the element is assumed to remain constant after reaching its maximum—the angle of repose.

In a statically determinate beam under bending the limiting state of the structure as a whole is reached when, in one cross-section, all elements are stressed with σ_y (plastic hinge) whereas in an indeterminate one there must be enough plastic hinges to turn it into a mechanism. In the case where there are only single loads, the position of the hinges is usually obvious but that is not so in the case of distributed loads. The magnitude of the plastic moment, M_{pl}, however, depends only on σ_y and the cross-section. If, then, x_i denote the position of hinge "i," M_{pl} may be expressed, for equilibrium, as a function $M_{pl} = f(P, x_i)$, and the true positions make M_{pl} a maximum or, in turn, P a minimum:

$$\frac{dM_{pl}}{dx_i} = 0, \quad i = 1, 2, 3 \ldots \text{etc.} \quad \ldots \quad (1)$$

A similar condition holds for σ_y if the sections vary.

If, instead, x is only estimated, M_{pl} will only slightly decrease. Thus reasonable approximations may be obtained very easily. For example, for a beam clamped at one side, freely supported on the other, and equally loaded, the correct distance of the hinge from the free support is $0{\cdot}414l$. By choosing it at midspan, the moment is found to be only 3 per cent too small.

Inversely, if two cases of loading act simultaneously (for example, the former plus a single load at midspan) the true hinge will be somewhere between $0{\cdot}414l$ and $0{\cdot}5l$. If, then, the moments corresponding to each case are added, a value is obtained which is slightly more than the correct value (the modified principle of superposition).

The foregoing holds also for plates (K. W. Johanson's theory). The

† The full MS. and illustrations may be seen in the Institution Library.—Sec. I.C.E.

parameters of the network of fracture lines are either to be rigidly determined by equation (1) or estimated.

Since the influence of the correct position of the hinges is small, in tests with beams, frames, or plates the ultimate load will often be found according to the plastic theory but the collapse mode will not.

In soil mechanics, too, the sliding surfaces can be determined either by a condition similar to equation (1) or by estimation. In a rather complex earth-pressure problem the Author has found an error of only 7 per cent by adopting a sliding plane inclined as 60 degrees instead of 52·5 degrees as obtained by a maximum condition.

The Paper is accompanied by two sheets of diagrams.

CORRIGENDA

Journal, November, 1951

Paper No. 5820 (Carroll)

In *Fig. 2*, p. 497, *for* "5·5 per cent" *read* "15·5 per cent."

Supplement, October, 1951

Correspondence on Paper No. 5801 (Neal and Symonds)

p. xxxvi, line 16, *for* "moment" *read* "stress"
line 18, *after* "loading" *add* "for shakedown to occur."

p. xxxvii, at end of Author's reply, *add*, "Moreover, in the case of proportional loading, the greatest elastic stresses do not occur at the three cross-sections 2, 3, and 4."

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